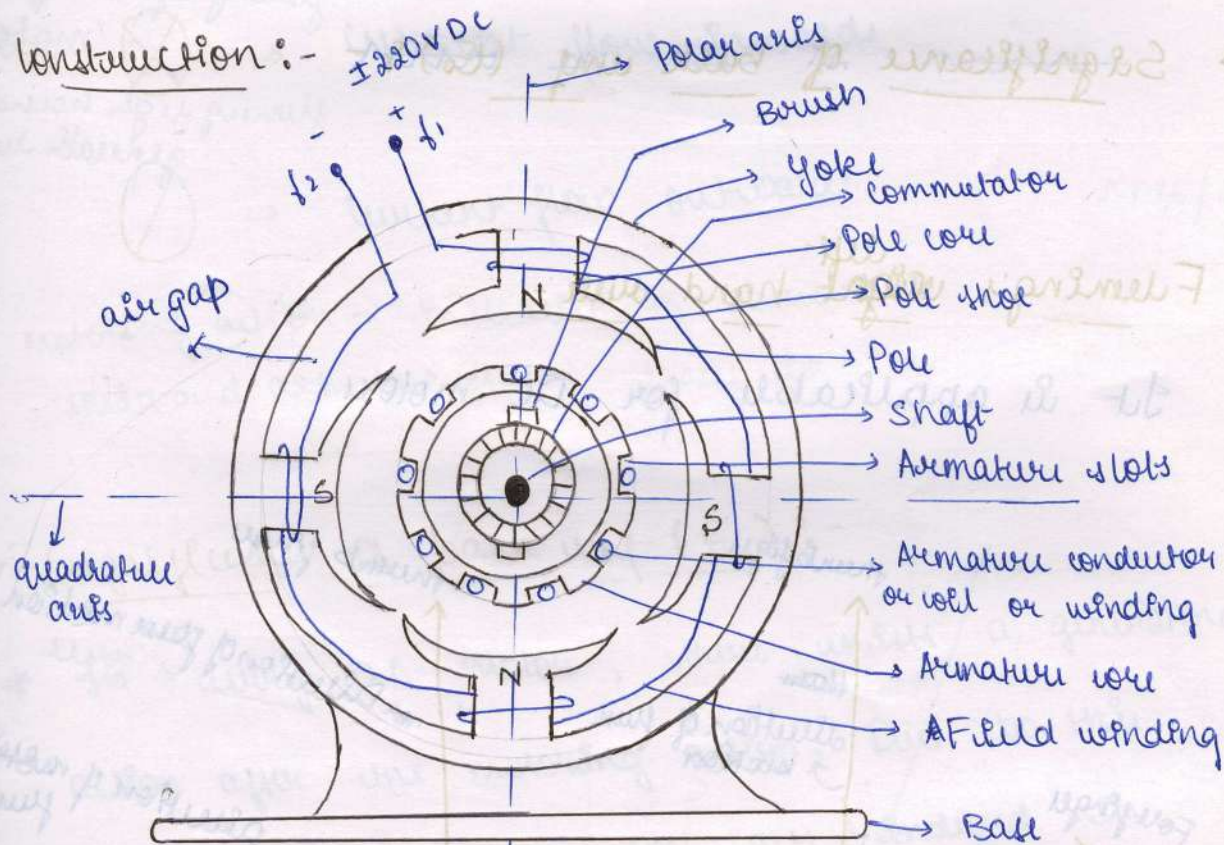


MODULE-1

DC - MOTORS

DC - motor :-

Construction :-



Principle :- DC motor

* whenever a current carrying conductor / coil is placed in a magnetic field, it experiences a mechanical force.

* The magnitude of ~~the~~ force experienced is given by

$$F = B I l \quad \text{Newtons}$$

where B = Magnetic flux density - Wb/m^2

I = current flowing through coil/conductor - Amp

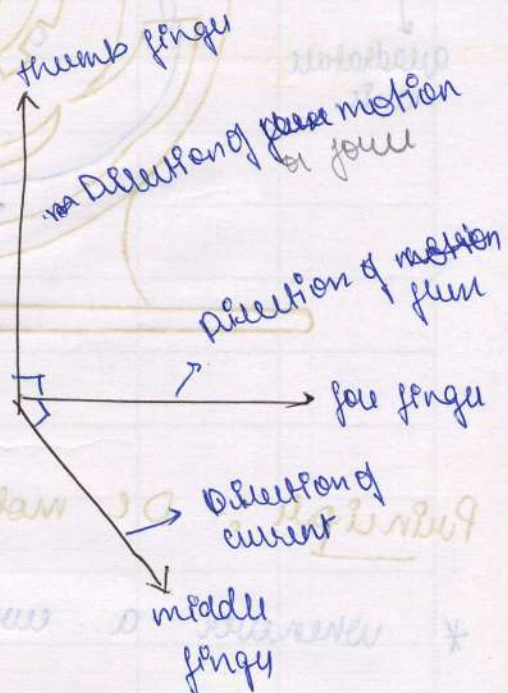
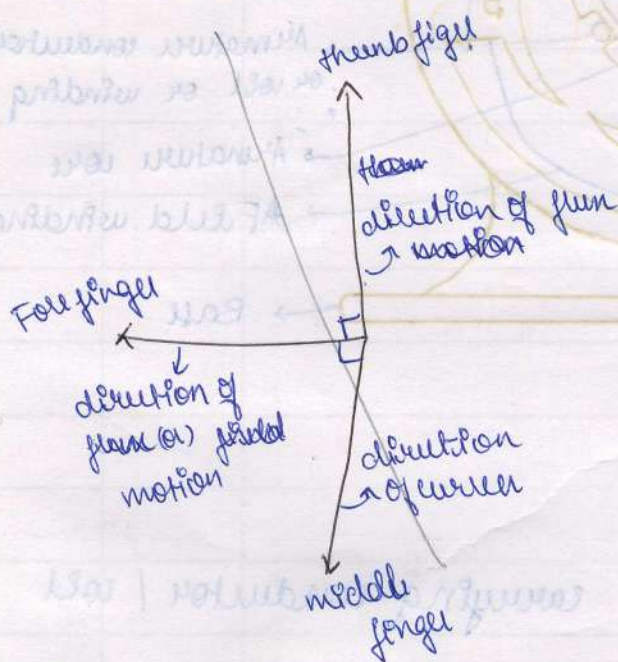
l = Active length of conductor

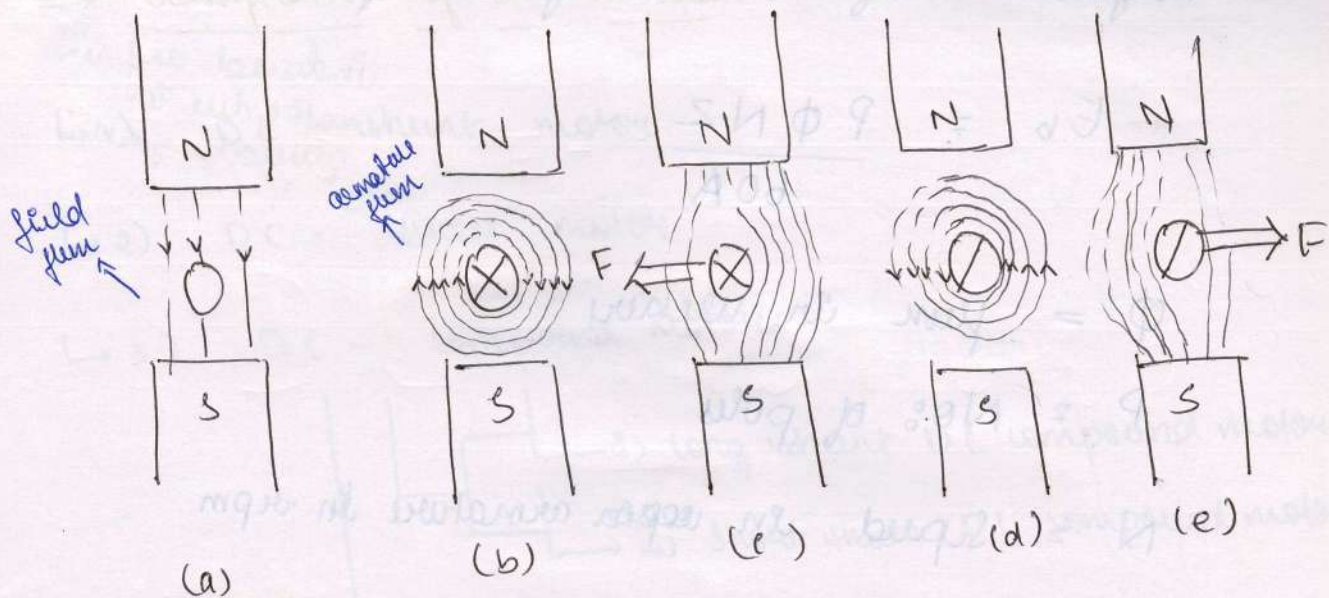
* Significance of back and motor

Fleming left hand rule
 Fleming right hand rule

Fleming's left hand rule -

It is applicable for DC motor





field

 $\rightarrow$ current flow inwards

 $\rightarrow$ current flow outwards

7/10/2019

motoring action - interaction of fluxes -
field and armature fluxes leads to

$\Rightarrow$ Significance of back emf (E_b) :-

- * In every DC motor, there exists a generating action after the motoring action. Due to this generating action, an emf gets induced in the armature winding / conductor.
- * This induced emf opposes the supply voltage V . due to Lenz's law and is known as the back emf E_b .

* Magnitude of E_b is given by

$$E_b = \frac{P \phi N Z}{60 A}$$

induced e.m.f. in motor due to generating action

ϕ = flux in webers

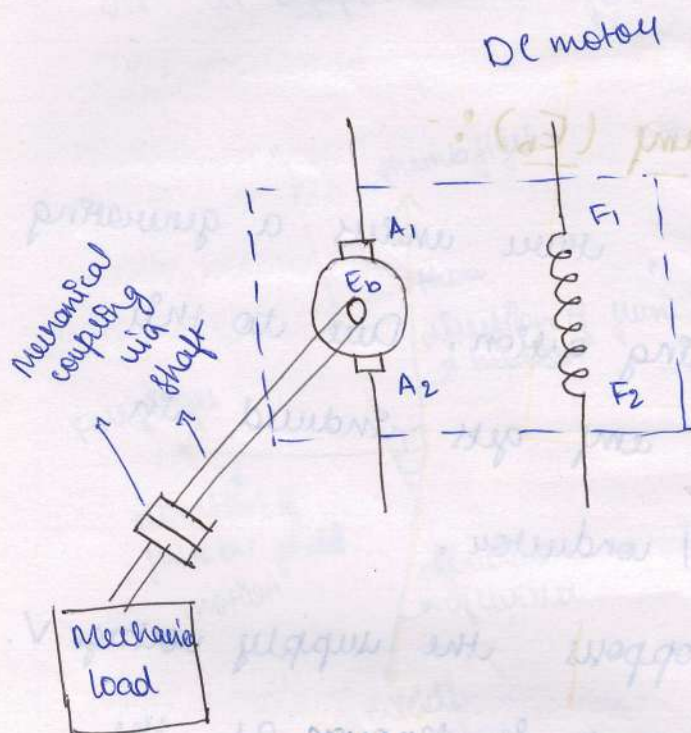
P = No. of poles

N = Speed in rpm

Z = Armature conductors

A = No. of parallel paths

⇒ Symbolic representation of a DC motor :-

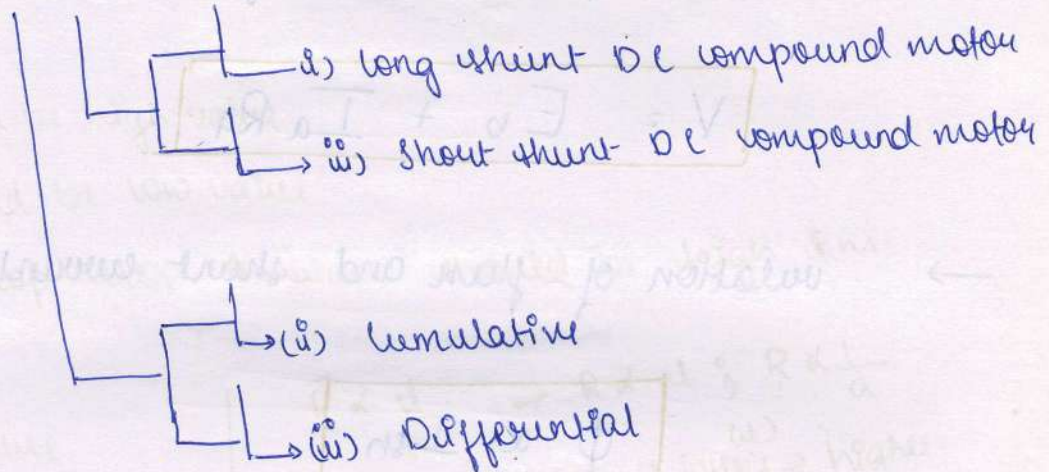


⇒ Classification of DC - motors:-

↳ 1) DC - shunt motor

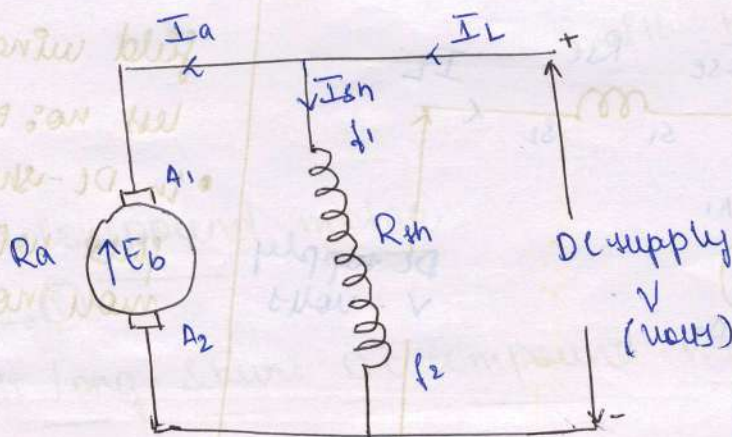
↳ 2) DC - series motor

↳ 3) DC - compound motor



↳ 1) DC - shunt motor:-

I_L → line current



→ current relationship

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

→ Voltage equation

$$V = E_b + I_a R_a + V_{brush}$$

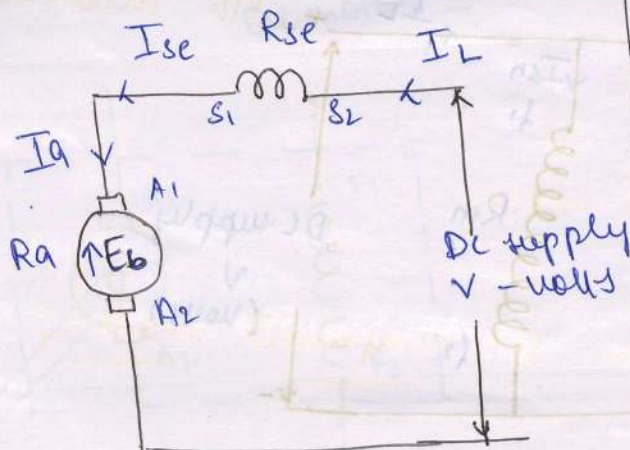
Generally V_{brush} is neglected

$$V = E_b + I_a R_a$$

→ relation of flux and shunt current

$$\phi \propto I_{sh}$$

→ 2) DC - series motor



• In DC-series motor field winding has less no. of turns

• In DC-shunt motor field winding has more no. of turns

→ current relationship

$$I_L = I_{se} = I_a$$

→ relation of flux and series current and line current

$$\phi \propto I_{se} \propto I_L \propto I_a$$

→ Voltage equation

$$\Rightarrow V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se}) \quad \text{neglecting } V_{brush}$$

R_{sh} should be high value

R_{se} should be low value

Voltage drop across Armature should be high & in series

$R_a \rightarrow$ low value

$R_{se} \rightarrow$ low value

$R_{sh} \rightarrow$ high value

$R \propto \frac{l}{a} \Rightarrow R \propto l ; R \propto \frac{1}{a}$
∴ R_{sh} - more no. of turns $\Rightarrow$ higher value
with thin cross section (area)

R_{se} - less no. of turns $\Rightarrow$ lower value
(1) with thick value (area)

→ 3) Compound motor

→ ①

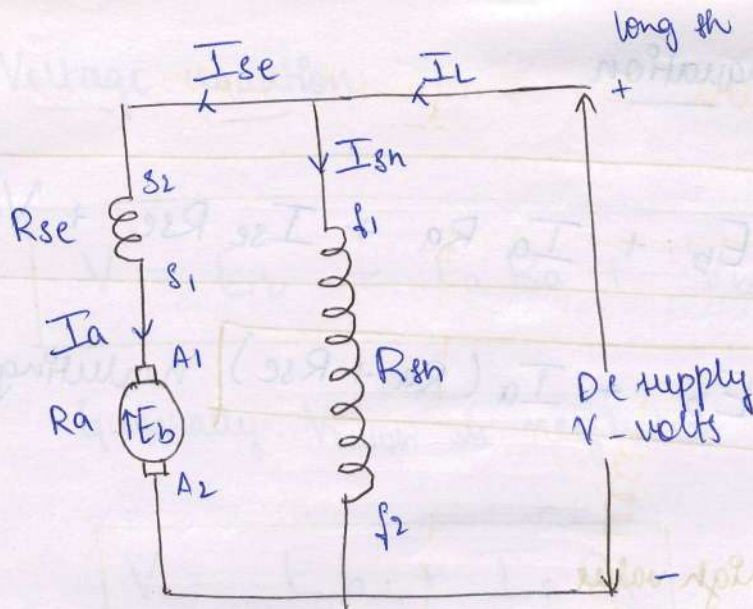
→ (ii) Long shunt DC compound motor :-

Types of winding

1) Armature

2) shunt field winding

3) series field winding



→ current relationship

$$I_L = I_{se} + I_{sh}$$

$$I_{se} = I_a$$

$$(or) I_L = I_a + I_{sh}$$

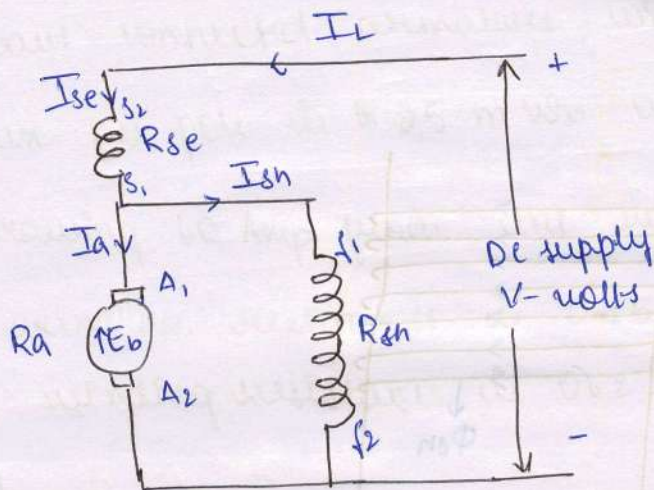
→ voltage equation

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se})$$

neglecting V_{brush}

Q.ii) Shunt shunt DC compound motor



→ Current relationship

$$I_L = I_a + I_{sh}$$

$$I_L = I_{se}$$

$$I_{se} = I_a + I_{sh}$$

→ Voltage equation

$$V = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$$

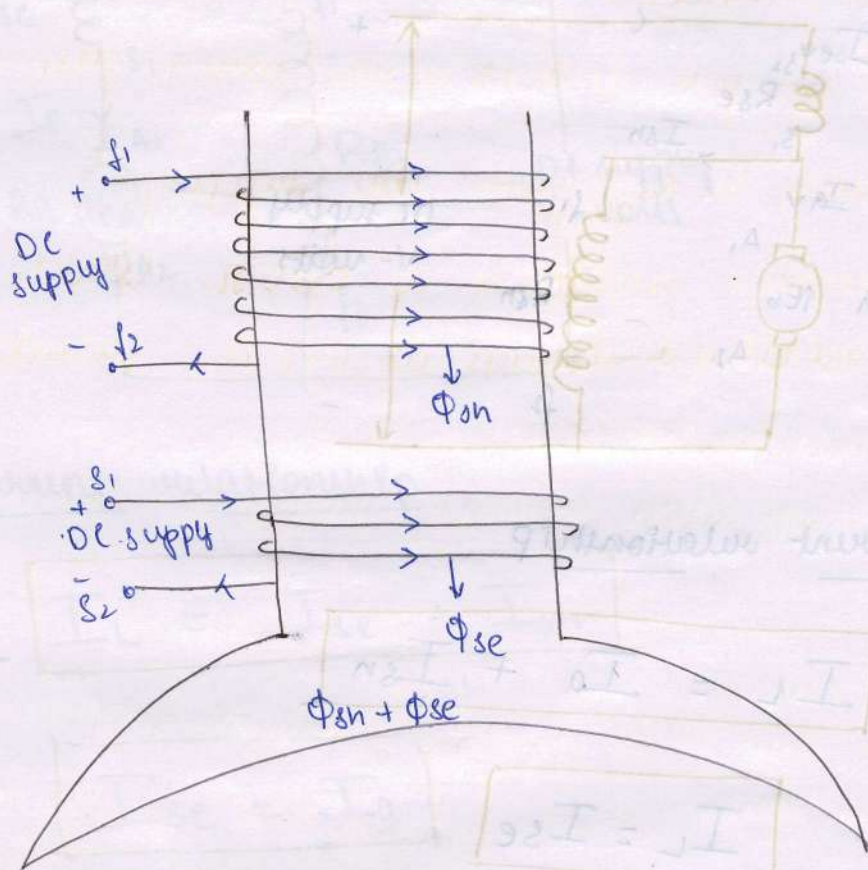
$$V = E_b + I_{se} R_{se} + I_a R_a + \text{neglecting } V_{brush}$$

⇒ Shunt current equation

$$I_{sh} = \frac{V - I_{se} R_{se}}{R_{sh}} = \frac{E_b + I_a R_a}{R_{sh}}$$

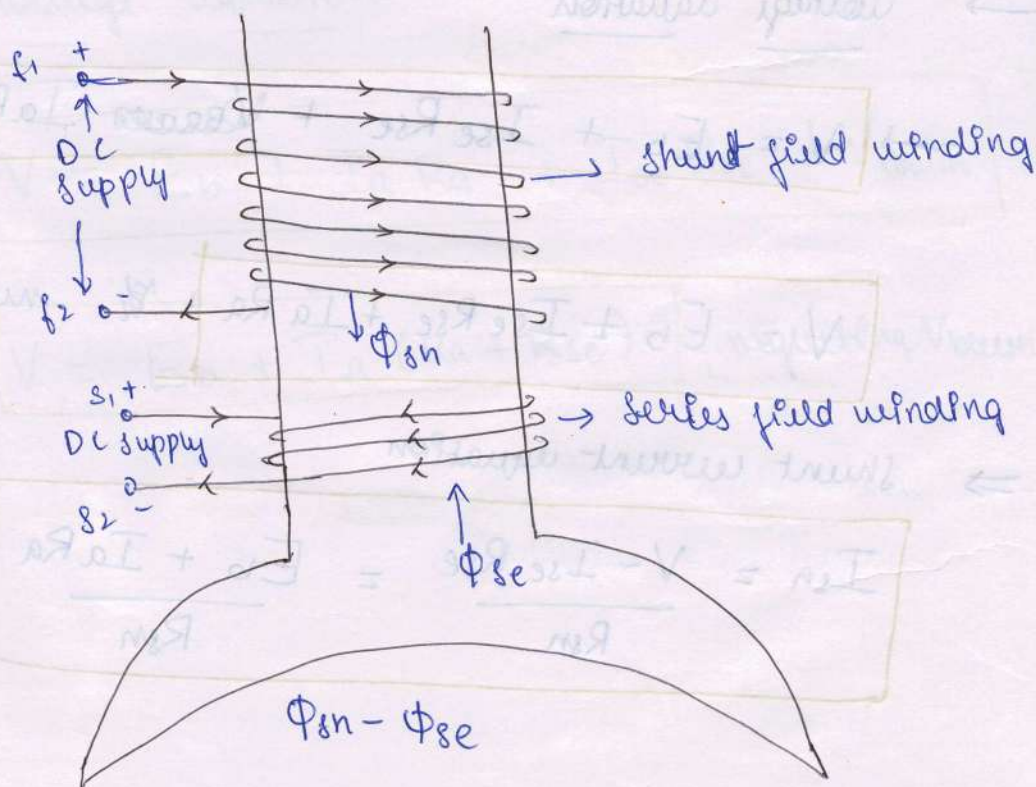
$\rightarrow \rightarrow \textcircled{ii}$
 $\rightarrow$ (i) Cumulative compound DC motor:-

Q1)



Soln

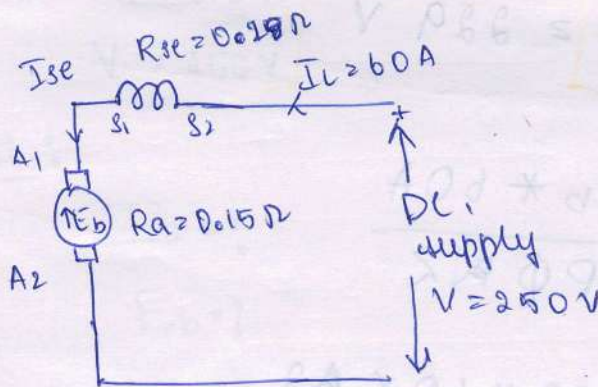
$\rightarrow \textcircled{iii}$ Differential compound DC motor:-



Q1) A 4 pole, 250 V, DC series motor has a wave connected armature with 200 conductors. A flux per pole is 25 mWb when the motor is drawing 60 Amp from the supply.

The armature resistance is 0.15Ω while series field winding resistance is 0.2Ω . Calculate the speed.

Soln



Given data:-

$$P = 4$$

$$\Phi = 25 \times 10^{-3} \text{ Wb}$$

$$Z = 200$$

$$R_a = 0.15 \Omega$$

$$R_{se} = 0.2 \Omega$$

$$I_L = 60 \text{ A}$$

$$V = 250 \text{ V}$$

$$A = 2$$

To find:-

$$N = ?$$

* Motor takes a current
(or)
Motor draws a current-

⇒ ~~IL~~ means line

current I_L not I_a

$$\rightarrow E_b = \frac{P \Phi N Z}{60A}$$

$$= \frac{40 \times 25 \times 10^3}{60 \times 2}$$

$$V = E_b + I_{se} R_{se} + I_a R_a$$

$$250 = E_b + 60 \times 0.2 + 60 \times 0.15$$

$$E_b = 229 \text{ V}$$

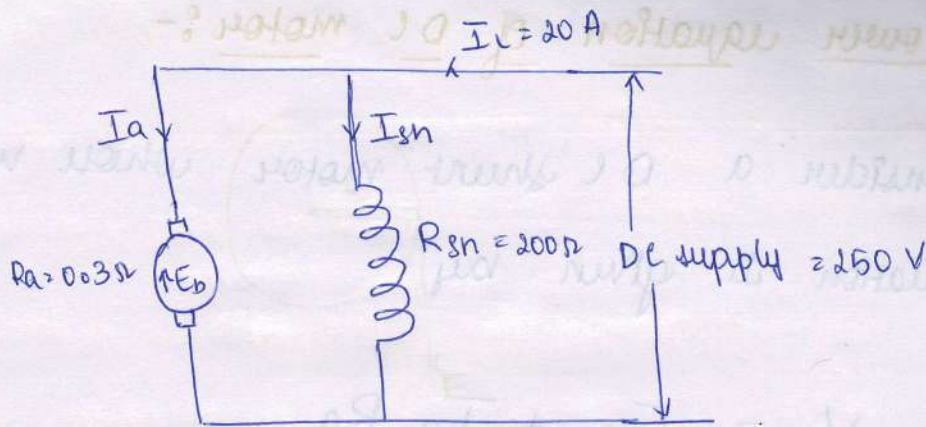
$$\rightarrow N = \frac{E_b \times 60A}{P \Phi Z}$$

$$= \frac{229 \times 60 \times 2}{4 \times 25 \times 10^{-3} \times 200}$$

$$N = \underline{\underline{1374 \text{ rpm}}}$$

Q2) A 250 V, DC shunt motor takes a line current of 20 Amps. The resistance of shunt field winding is 200 Ω and resistance of armature is 0.3 Ω . Find the armature current and back emf

Soln :-



Given data :-

$$I_L = 20A$$

$$R_{sh} = 200\Omega$$

$$R_a = 0.03\Omega$$

$$V = 250V$$

To find :-

$$I_a = ?$$

$$E_b = ?$$

Soln :-

$$I_L = I_a + I_{sh} \rightarrow I_a = I_L - I_{sh} \rightarrow (1)$$

$$I_{sh} = I_L - I_a \rightarrow (2)$$

$$V = I_{sh} R_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{250}{200}$$

$$I_{sh} = 1.25 \text{ Amps} \rightarrow (2)$$

Sub (2) in (1)

$$I_a = I_L - I_{sh} = 20 - 1.25 = 18.75 \text{ Amps}$$

$$V = E_b + I_a R_a$$

$$E_b = 250 - 18.75 \times 0.03$$

$$E_b = 244.375 V$$

// $(O/P \geq I/P - \text{losses})$ // $(I/P \geq O/P + \text{losses})$
 $\Rightarrow$ Power equation of DC motor:-

Consider a DC shunt motor whose voltage equation is given by

$$V = E_b + I_a R_a$$

Multiply both sides by current I_a

$$V I_a = E_b I_a + I_a^2 R_a$$

Power eqn of DC motor

where $V I_a$ = Power input to the armature in watts

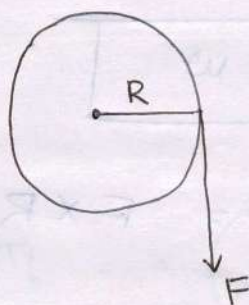
$I_a^2 R_a$ = Power loss in armature in watts

$E_b I_a$ = Electrical equivalent of gross mechanical power developed (power output)

$\Rightarrow$ Torque equation of a DC motor:-

The turning and twisting movement of force about a particular axis is called torque.

Consider a wheel of radius ' R '-metres acted upon by a circumferential force ' F '-newtons



Let the wheel rotate at 'N' rpm

i) The angular speed is given by

$$\omega = \frac{2\pi N}{60} \text{ rad/sec}$$

Power developed

ii) Work done is given by

$$W = \text{Force} \times \text{distance travelled}$$

$$= F \times 2\pi R \quad \text{N-m}$$

iii) Power developed is given by -

$$P = \frac{\text{work done}}{\text{Time taken (in sec)}}$$

$$P = \frac{F \times 2\pi R}{\left(\frac{60}{N}\right)}$$

$$P = (F \times R) \left(\frac{2\pi N}{60} \right)$$

$$P = (F \times R) \times \omega$$

$$\boxed{P = T \times \omega}$$

where torque $T = F \times R$

when the speed of ~~rpm~~ the armature is
'N' rpm

$E_b I_a$ = Electrical equivalent of
mechanical power developed
in armature.

Let T_a = the torque developed in the
armature

$$\therefore P = T_a \times \omega$$

$$E_b I_a = T_a \times \left(\frac{2\pi N}{60} \right)$$

Table

$\omega \cdot \text{kt}$

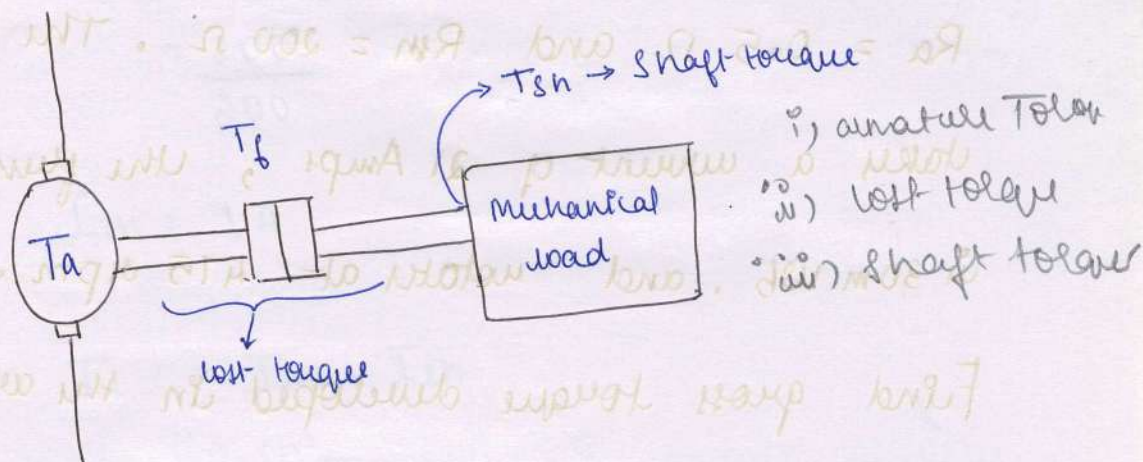
$$E_b = \frac{P \Phi N Z}{60 A}$$

$$\frac{P \Phi N Z}{60 A} I_a = T_a \times \frac{2\pi N}{60}$$

$$T_a = \frac{1}{2\pi} \frac{\Phi P Z}{A} I_a = \underline{\underline{0.159 \frac{P \Phi Z I_a}{A}}}$$

$$\boxed{T_a = \frac{1}{2\pi} \frac{P \Phi Z I_a}{A}} \quad - \text{Nm}$$

⇒ Types of Torque in a DC motor:-



- The entire armature torque (T_a) developed in the armature is not transmitted to the load because there are some losses i.e., friction and windage losses at the shaft (coupling) known as lost torque - T_l
- The useful torque available to the mechanical load at the end of the shaft is called shaft torque - T_{sh}

$$\boxed{T_{sh} < T_a} \rightarrow [\text{Due to losses}]$$

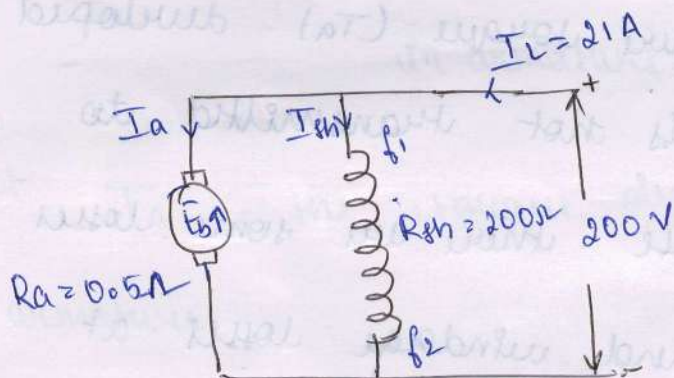
(i) Armature torque

(ii) lost torque $\rightarrow$ Friction loss
 $\rightarrow$ windage loss

(iii) shaft torque

Q) A 200V, 4 pole, lap wound DC shunt motor has 800 armature conductors. $R_a = 0.5 \Omega$ and $R_{sh} = 200 \Omega$. The motor takes a current of 21 Amps, the flux/pole is 30 mWb, and rotates at 475 rpm. Find gross torque developed in the armature.

Soln



Given:-

$$\phi = 30 \times 10^{-3} \text{ Wb}$$

$$N = 475 \text{ rpm}$$

$$\text{Lap} = A = P = 4$$

$$Z = 800$$

To find:- $T_a = ?$

$$T_a = \frac{P \phi Z I_a}{2 \pi A}$$

$$I_L = I_a + I_{sh}$$

$$\rightarrow I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{200}{200}$$

$$I_{sh} = 1 \text{ A}$$

$$\rightarrow I_L = I_a + I_{sh}$$

$$21 - 1 = I_a$$

$$I_a = 20 \text{ A}$$

$$T_a = \frac{P \phi Z I_a}{2\pi A}$$

$$= \frac{4 \times 30 \times 10^{-3} \times 800 \times 20}{2\pi \times 4}$$

$$T_a = \underline{\underline{76.039 \text{ N-m}}}$$

⇒ Speed and Torque equations :-

W. K T General torque expression is -

$$\rightarrow T_a = \frac{1}{2\pi A} * P \phi Z I_a \quad \text{N-m}$$

$$\boxed{T_a \propto \phi I_a} \rightarrow \textcircled{1}$$

because $\frac{1}{2\pi}$, P , A , Z are all constants

For all the

1) ⇒ For DC shunt motor :- (constant ϕ flux machines)
as I_a is constant $\rightarrow \phi$ is also constant

As long as the supply 'V' remains constant,

I_{sh} also remains constant and ϕ value remains constant ($\phi \propto I_{sh}$)

|| ⇒ Thus DC shunt motors are also called constant flux machines

$$\boxed{T_a \propto I_a}$$

Application :-

→ Used for moderate starting torque applications

2) $\Rightarrow$ For DC series motor :-

WKT

$$I_L = I_{se} = I_a$$

Now $\phi \propto I_{se}$

$$\Rightarrow \phi \propto I_a$$

$$T_a \propto \phi I_a$$

from eqn (1)

$$T_a \propto I_a \times I_a$$

$$\boxed{T_a \propto I_a^2}$$

$\Rightarrow$ Speed equation :-

WKT

$$E_b = \frac{\phi P N Z}{60 A} \text{ Volts}$$

$$\boxed{E_b \propto \phi N}$$

because $P, Z, 60, A$ are all constants

// $\therefore \Rightarrow$

$$\boxed{N \propto \frac{E_b}{\phi}}$$

$\rightarrow (2)$

⇒ For DC shunt motors:-

w.k.T ϕ remains constant

$$N \propto E_b$$

$$\therefore V = E_b + I_a R_a$$

from voltage equations

$$N \propto (V - I_a R_a)$$

⇒ For DC series motors:-

$$N \propto \frac{E_b}{I_a}$$

from eqn (2)

$$\phi \propto I_a$$

$$V = E_b + I_a R_a + I_s R_s$$

From voltage equations

$$N \propto \left[\frac{V - I_a R_a - I_s R_s}{I_a} \right]$$

$$N \propto \frac{E_b}{\phi}$$

⇒ Characteristics of DC motors :-

- 1) Torque versus armature current (T_a vs I_a) :-
 ↳ Electrical characteristics (also called)
- 2) Speed versus armature current (N vs I_a)
- 3) Speed versus Torque (N vs T_a)
 ↳ Mechanical characteristics (also called)

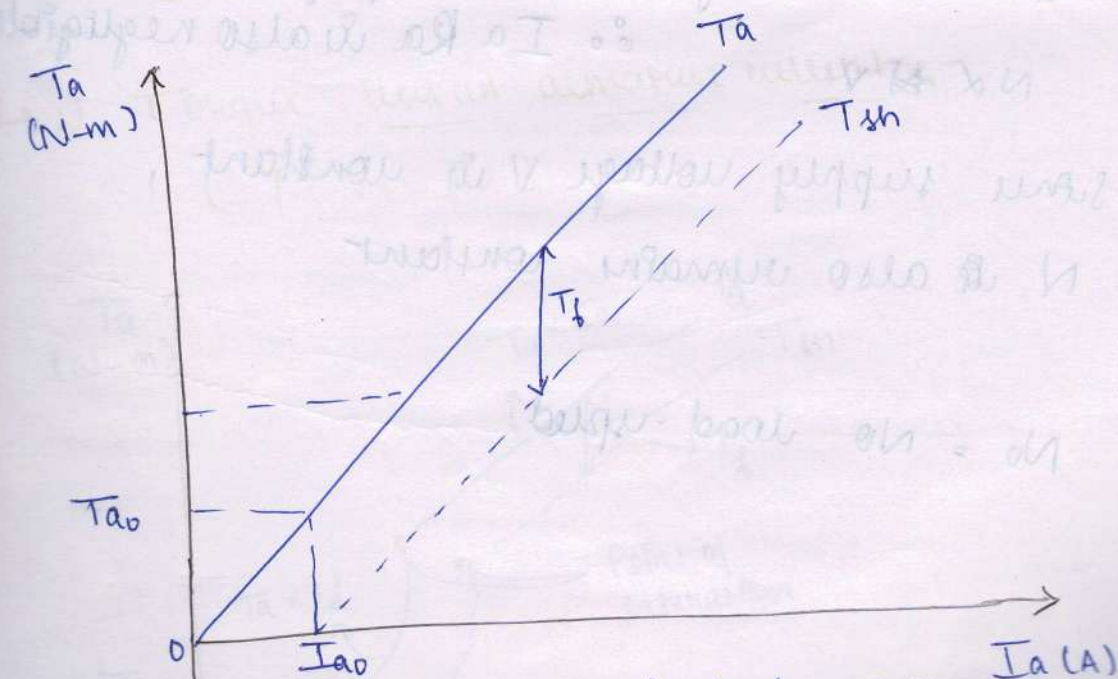
↳ Characteristics of DC shunt motor :-

- 1) Torque versus armature current :-

T_a vs I_a
 W.K.T

$T_a \propto I_a$

Shunt motor can be started on no load
 • Series motor requires load to start



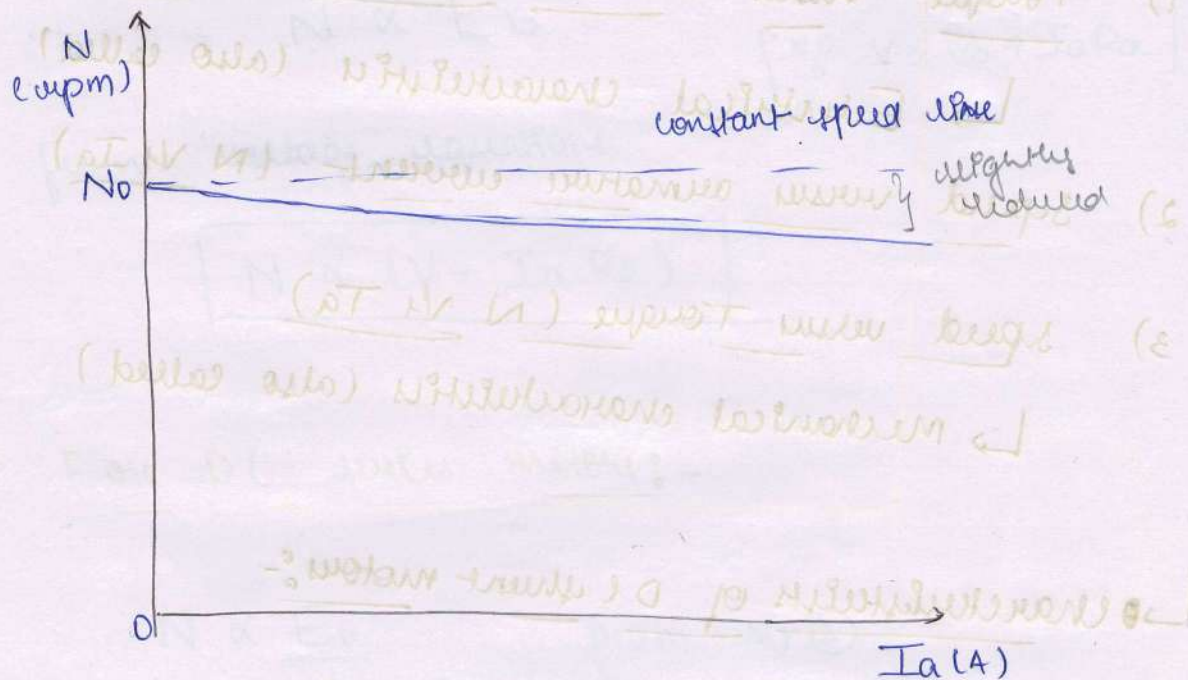
T_a - armature torque
 T_{sh} - shaft torque

$T_{a0} \rightarrow$ No load armature torque

$I_{a0} \rightarrow$ No load armature current

L2) Speed versus armature current :-

(N vs I_a)



$$N \propto \frac{E_b}{\phi}$$

$$N \propto (V - I_a R_a)$$

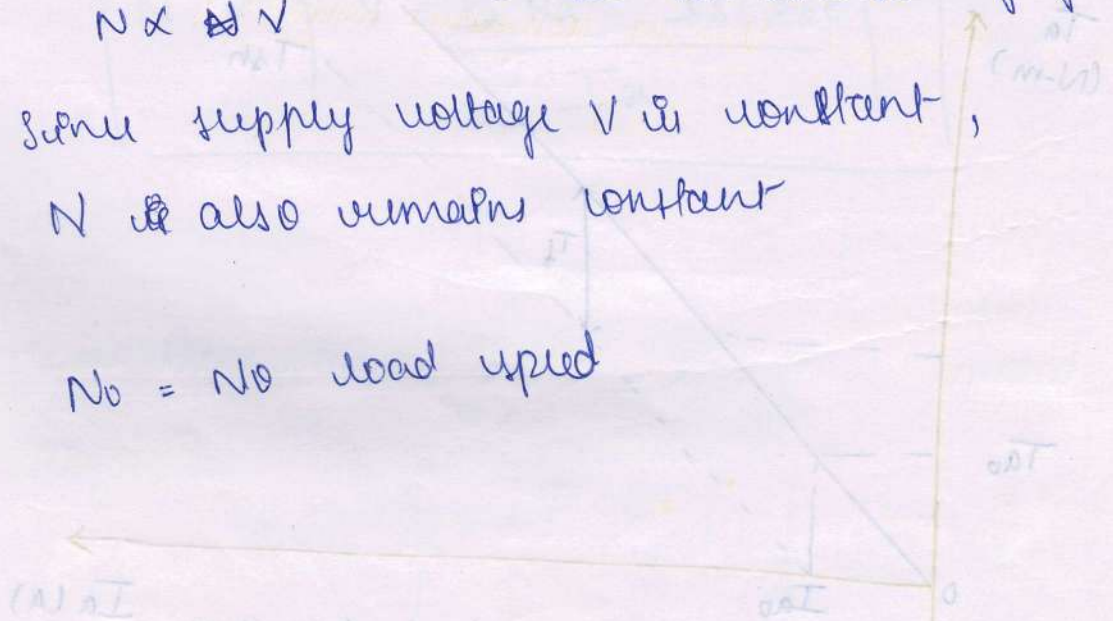
$R_a \rightarrow$ is very small, negligible
 $\therefore I_a R_a$ is also negligible

$$N \propto V$$

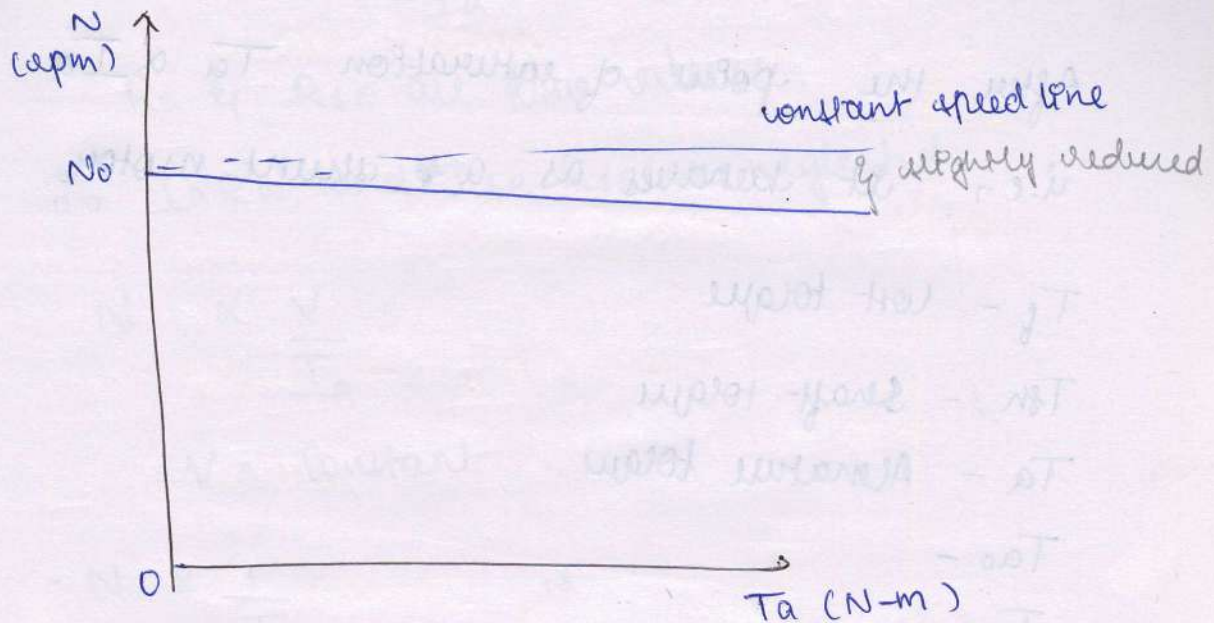
Since supply voltage V is constant,

N also remains constant

$N_0 = N$ load speed



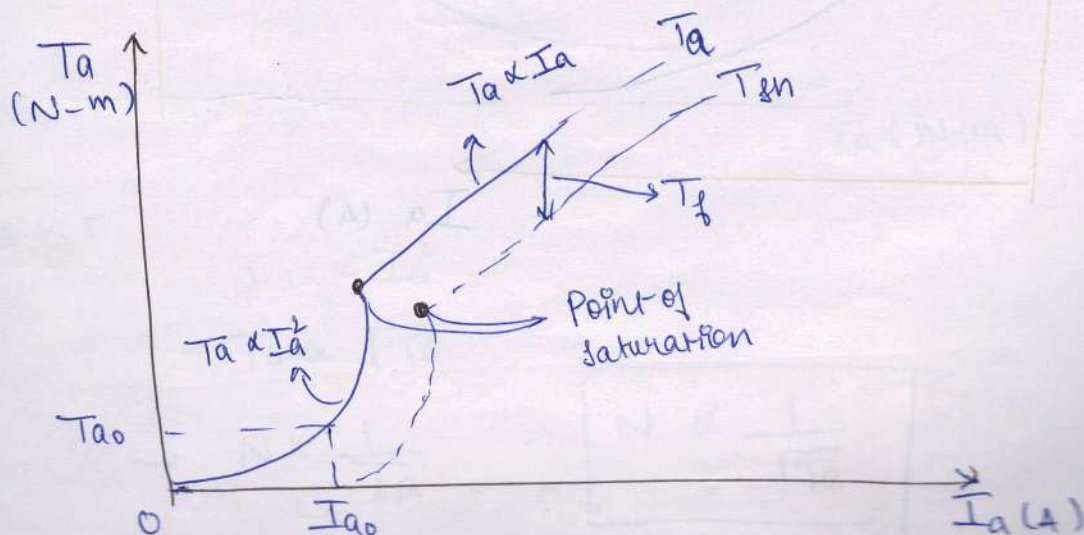
→ 3) Speed versus Torque :-
($N \propto T_a$)



Since $T_a \propto I_a$
(Graph of N vs T_a is similar to N vs I_a characteristics)

→ 4) characteristics of DC series motor :-

→ 1) Torque versus armature current :-
($T \propto I_a$)



W. K. T

$$T_a \propto I_a^2$$

After the point of saturation $T_a \propto I_a$

i.e., it behaves as a shunt motor

T_f - field torque

T_{sh} - shaft torque

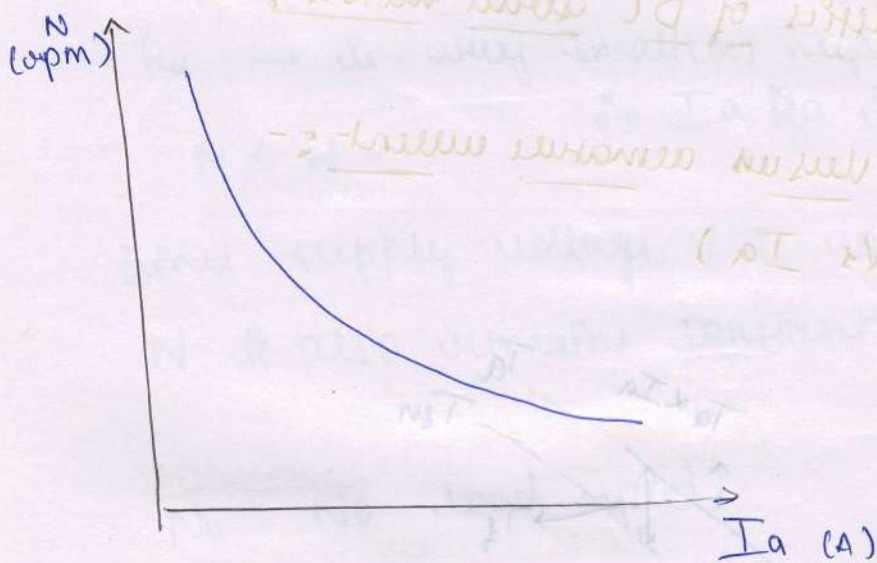
T_a - Armature torque

T_{ao} -

I_{ao} -

→ 2) Speed versus armature current

(N vs I_a)



$$N \propto \frac{E_b}{\phi}$$

$$N \propto \frac{V - I_a R_a - I_{se} R_{se}}{I_a}$$

R_a & R_{se} are known values

∴ $I_a R_a$ and $I_{se} R_{se}$ are neglected

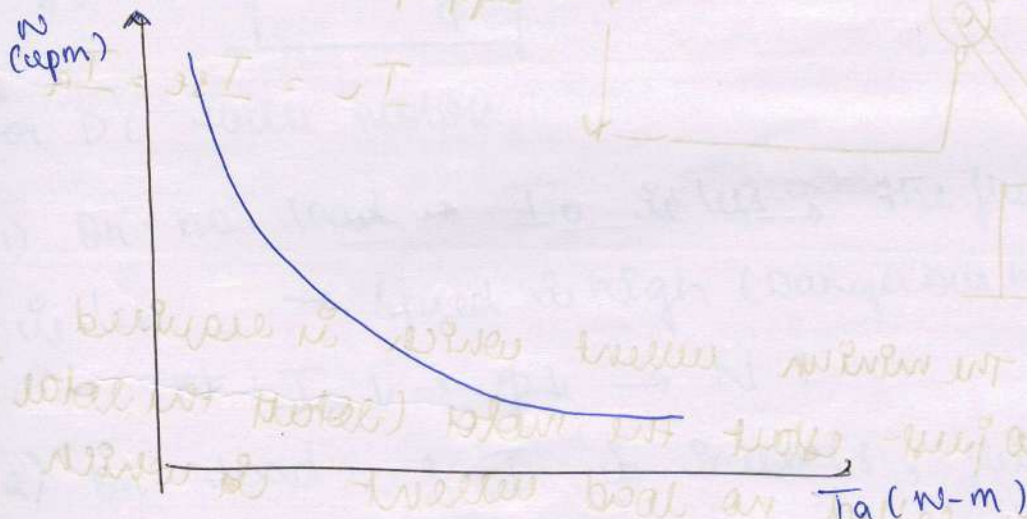
$$N \propto \frac{V}{I_a}$$

$V = \text{constant}$

$$N \propto \frac{1}{I_a}$$

↳ 3) Speed versus Torque :-

(N vs T_a)



$W \propto T$

$$T_a \propto I_a^2$$

$$I_a \propto \sqrt{T_a}$$

$$\rightarrow N \propto \frac{1}{I_a}$$

$$\Rightarrow \boxed{N \propto \frac{1}{\sqrt{T_a}}}$$

Application

Series motor are always started with load

$$N \propto \frac{E_b}{\Phi} \Rightarrow \text{general eqn}$$

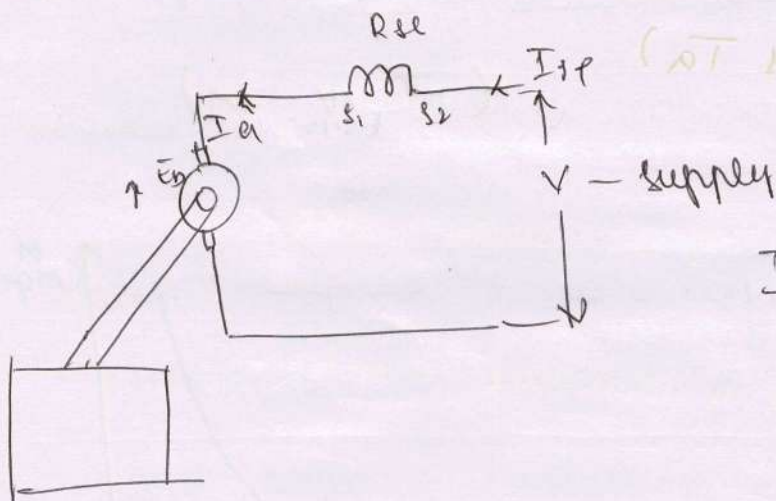
$$N \propto \frac{V - I_a R_a - I_{se} R_{se}}{\Phi}$$

$I_a R_a$; $I_{se} R_{se} \rightarrow \text{negligible}$

$$N \propto \frac{V}{\Phi}$$

[V - constant]

$$N \propto \frac{1}{\Phi}$$



$$I_a = I_{se} = I$$

* The minimum current which is required to just start the motor (start the motor) is called no load current I_{a0} which is very-very small

If I_{a0} - no load current is very small speed is high $\uparrow N \propto \frac{1}{\Phi} \propto \frac{1}{I_a} \downarrow \text{low}$

$I_a \downarrow, \Phi \downarrow \rightarrow N \uparrow$ (motor runs at high speed)

If there is load $\rightarrow$ current (no load)
(not-no-load)

I_a decrease and

current increase, then also increase,
speed decrease

$$I \uparrow, \phi \uparrow; N \downarrow \downarrow$$

DC series motor should never be
started on no-load.

$$\Rightarrow W.K.T$$

$$N \propto \frac{E_b}{\phi}$$

$$N \propto \frac{(V - I_a R_a - I_{se} R_{se})}{\phi}$$

$$N \propto \frac{1}{\phi}$$

(V is constant)

∴

$$\boxed{N \propto \frac{1}{\phi}}$$

For DC series motor

↳ 1) On no-load $\rightarrow I_a$ is less, the flux ϕ
is less, $\rightarrow$ speed is high (dangerous high speed)

$$I_a \downarrow \rightarrow \phi \downarrow \rightarrow N \uparrow$$

↳ 2) On load $\rightarrow I$ is increase, flux
increases and speed decreases

$$I \uparrow, \phi \uparrow; N \downarrow \downarrow$$

⇒ Applications of DC-motors:-

Type of motor	characteristics	Applications
DC shunt motor	- speed slightly constant and - medium starting torque	Fans, blowers, centrifugal pumps, lathe machines, Drilling machines, machines tools... etc
DC - series motor	- high starting torque, - variable speed - No load condition is dangerous	cranes, elevators, hoists, conveyor belts, electric locomotives and traction
Cumulative compound motor	- high starting torque, - No load condition is allowed	Rolling mills, elevators, lift... etc (For bidirectional application - lift)
Differential compound motor	speed increases as the load increases	Not suitable for any practical application

Differential compound motor

$$I_L = I_a + I_{sh}$$

$$\Phi_{sn} - \Phi_{se}$$

$$(\Phi_{sn} - \Phi_{se}) \downarrow$$

$$\uparrow N \propto \frac{E_b}{\Phi} \downarrow$$

In $\Phi_{se} \rightarrow$ I current is small $\rightarrow \Phi_{se}$ will be small
 Due to more Φ $\Phi_{se} \Rightarrow$ Difference of $(\Phi_{sn} - \Phi_{se})$

decreases
 due to speed increases

⇒ Factors affecting speed of a DC motor:-

WKT

$$N \propto \frac{E_b}{\Phi}$$

Say

$$N \propto \frac{V - I_a R_a}{\Phi}$$

$$\therefore \boxed{N \propto \frac{V}{\Phi}}$$

as $E_b \approx V$ [approximately equal to V]

Factors affecting the speed are -

↳ (i) Flux (Φ)

↳ (ii) Voltage across the armature

↳ (iii) supply voltage (V)

⇒ Depending on these factors, the various methods of speed control are -

↳ (i) Flux control method:-

By changing the flux Φ , by controlling the current through field winding

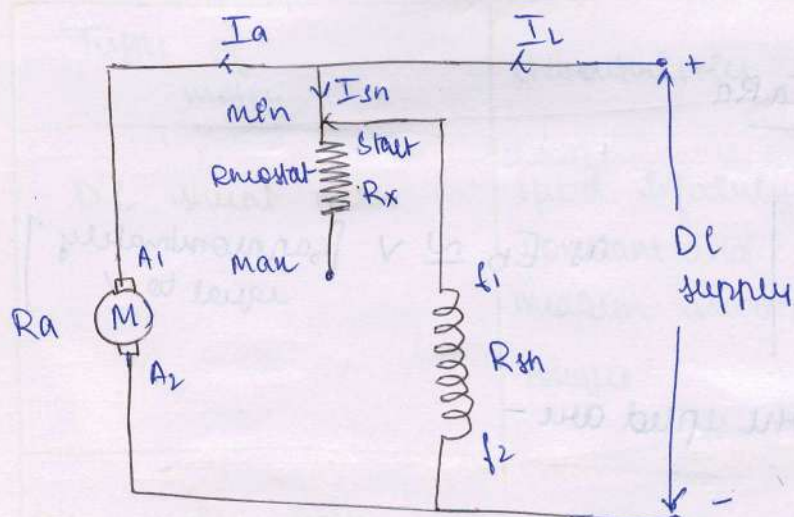
↳ (ii) Rheostatic control:- (or) Armature voltage control:-

changing the armature path resistance when it then changes the voltage across the armature.

↳ (iii) Voltage control method:- (or) ~~Reversal~~ ~~and~~ ~~reversal~~ ~~method~~:-
changing the applied voltage

⇒ Speed control of DC shunt motor :-

↳ (i) Field control method :-



As resistance ↑
through
Rheostat

I_{sh} decreases

∴ Φ_{sh} decreases

$$N \propto \frac{E_b}{\Phi}$$

$\Phi \downarrow$

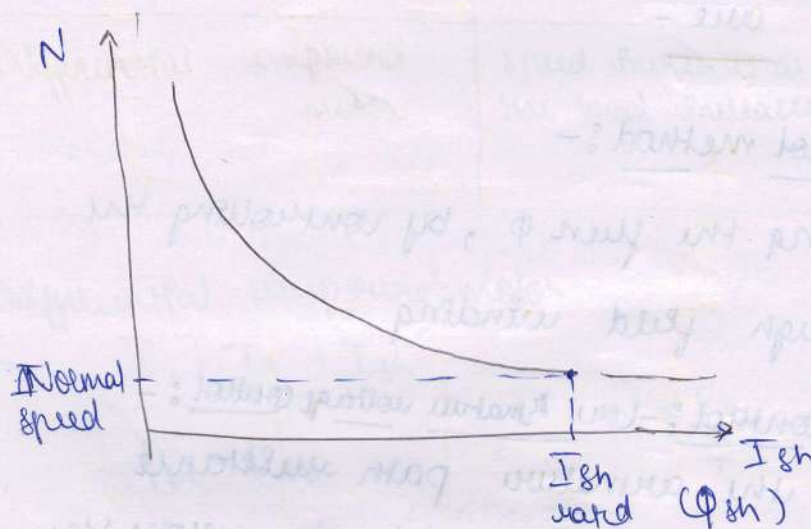
∴ N increases

Disadvantage

Speed cannot
be decreased in
this type of control
method

[i.e., below the
rated speed]

↳ N Vs I_{sh} :-



$$I_{sh} \propto \frac{V}{R_{sh}}$$

as $R_{sh} \uparrow \rightarrow I_{sh} \downarrow$

as $I_{sh} \downarrow \rightarrow \text{Speed } N \uparrow$

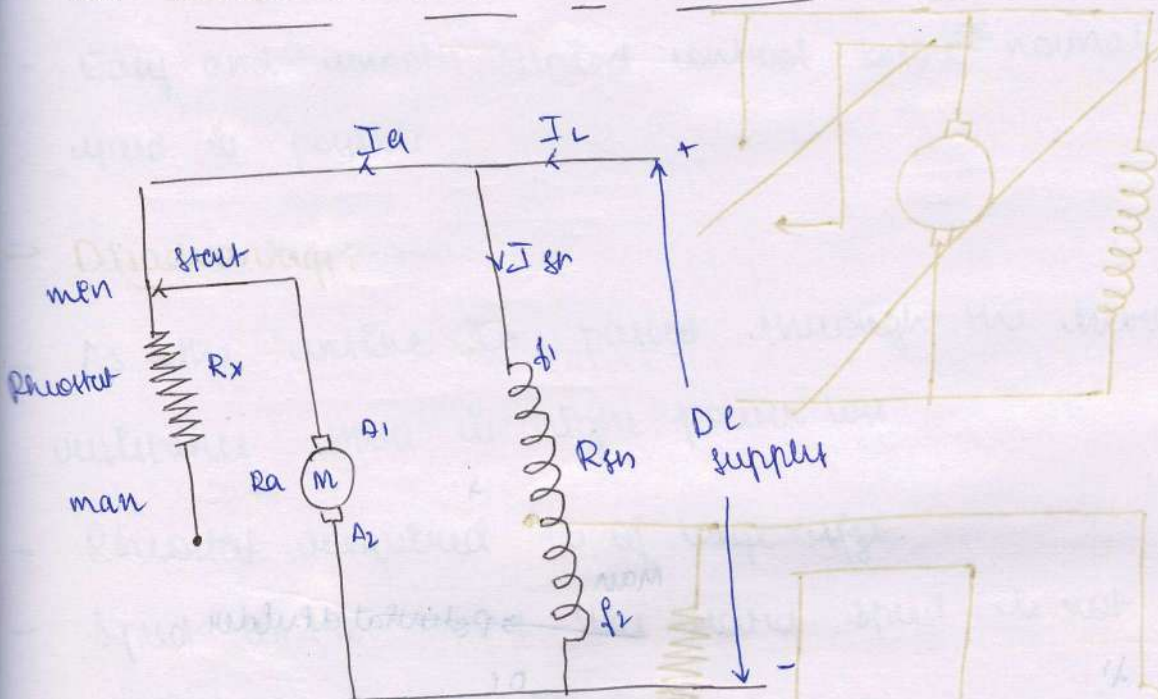
→ Advantages:-

- Speed control above rated speed is possible
 - It provides smooth and easy control.
 - As field current is low, the rheostat required is small.
- Thus the power loss in the rheostat is also less

→ Disadvantages:-

- Speed control below the normal rated speed is not possible

↳ (iii) Rheostatic control (or) Armature voltage control:-

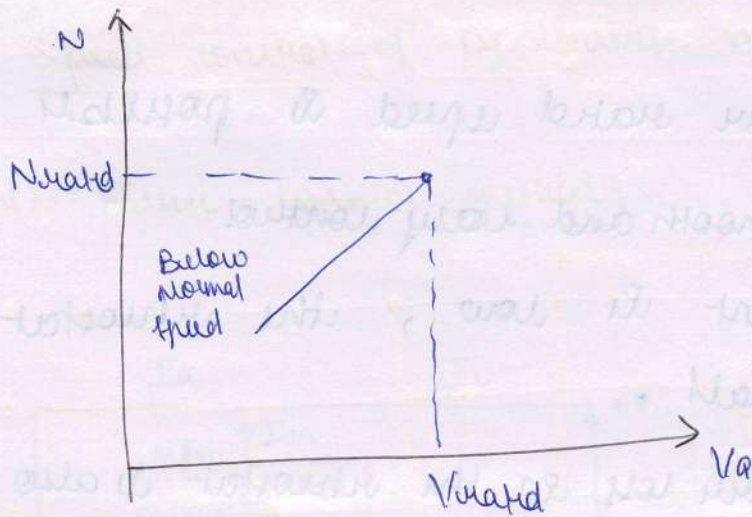


as $R_n \uparrow \rightarrow$ voltage drop across $I_a R_x$ increases $\uparrow$

$\rightarrow$ Due to $I_a R_a \rightarrow$ voltage drop decreases $\downarrow$

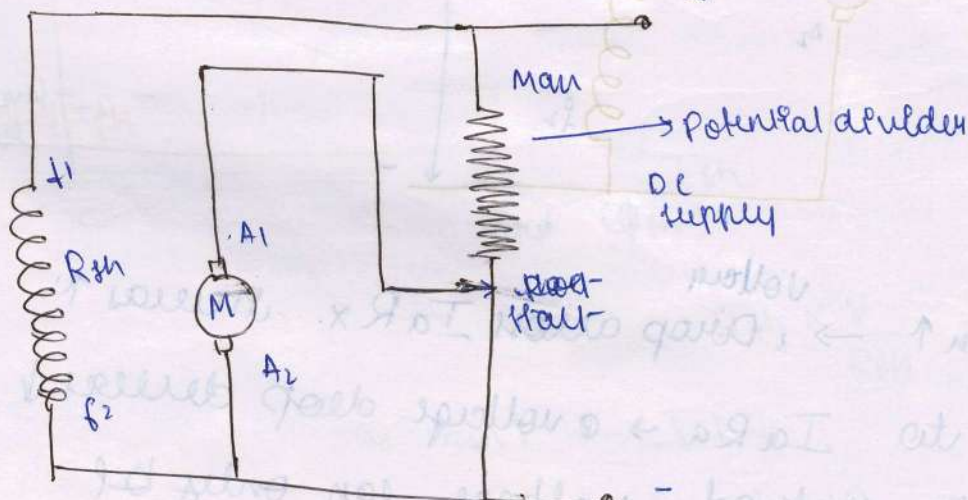
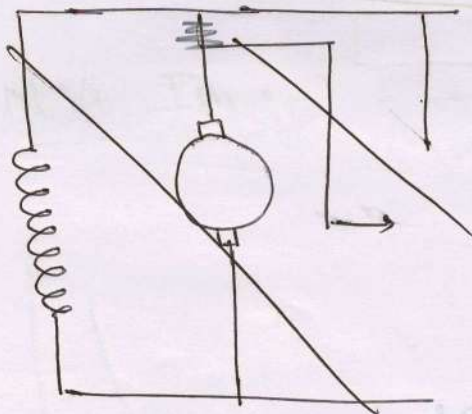
In this method voltage can only be decreased $\rightarrow$ ~~Armature~~ Voltage across armature is reduced.

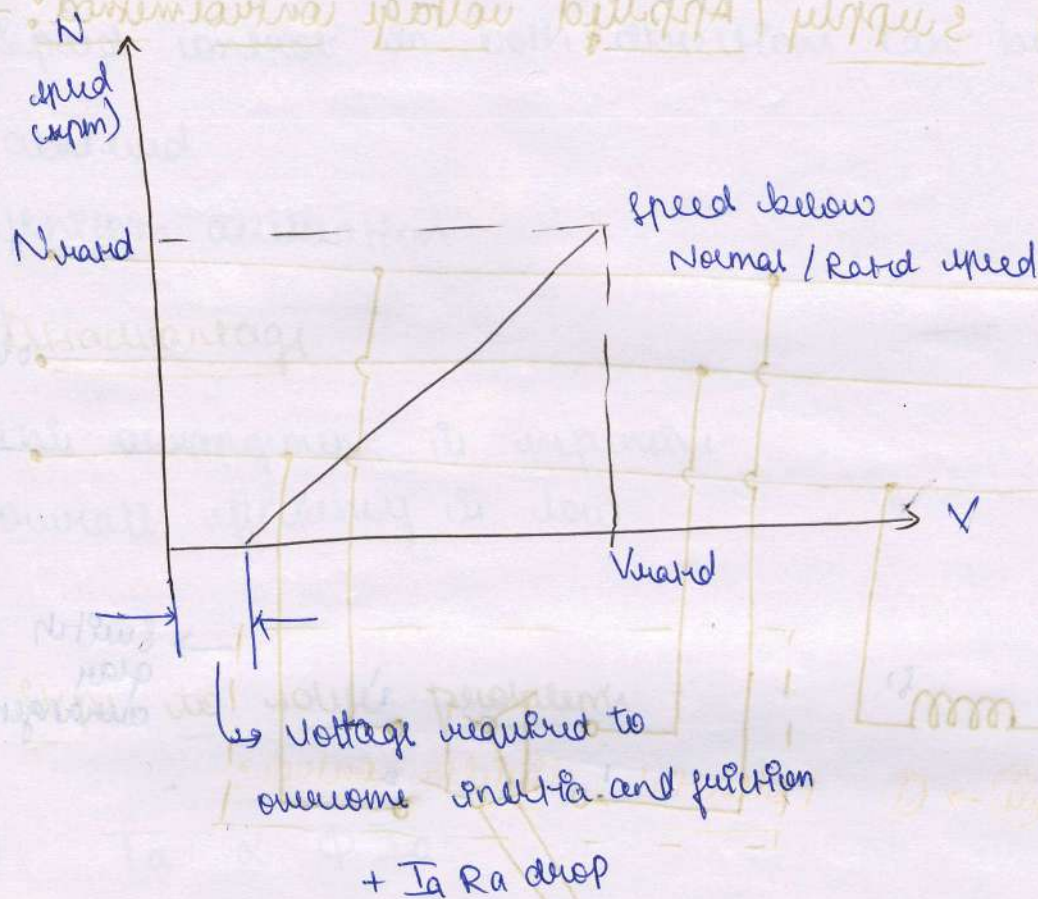
$\hookrightarrow N \quad V_s \quad V_a$



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$\hookrightarrow \textcircled{iii} \rightarrow \textcircled{i}$ Potential divider control:-





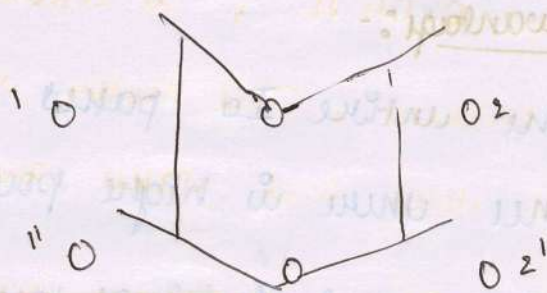
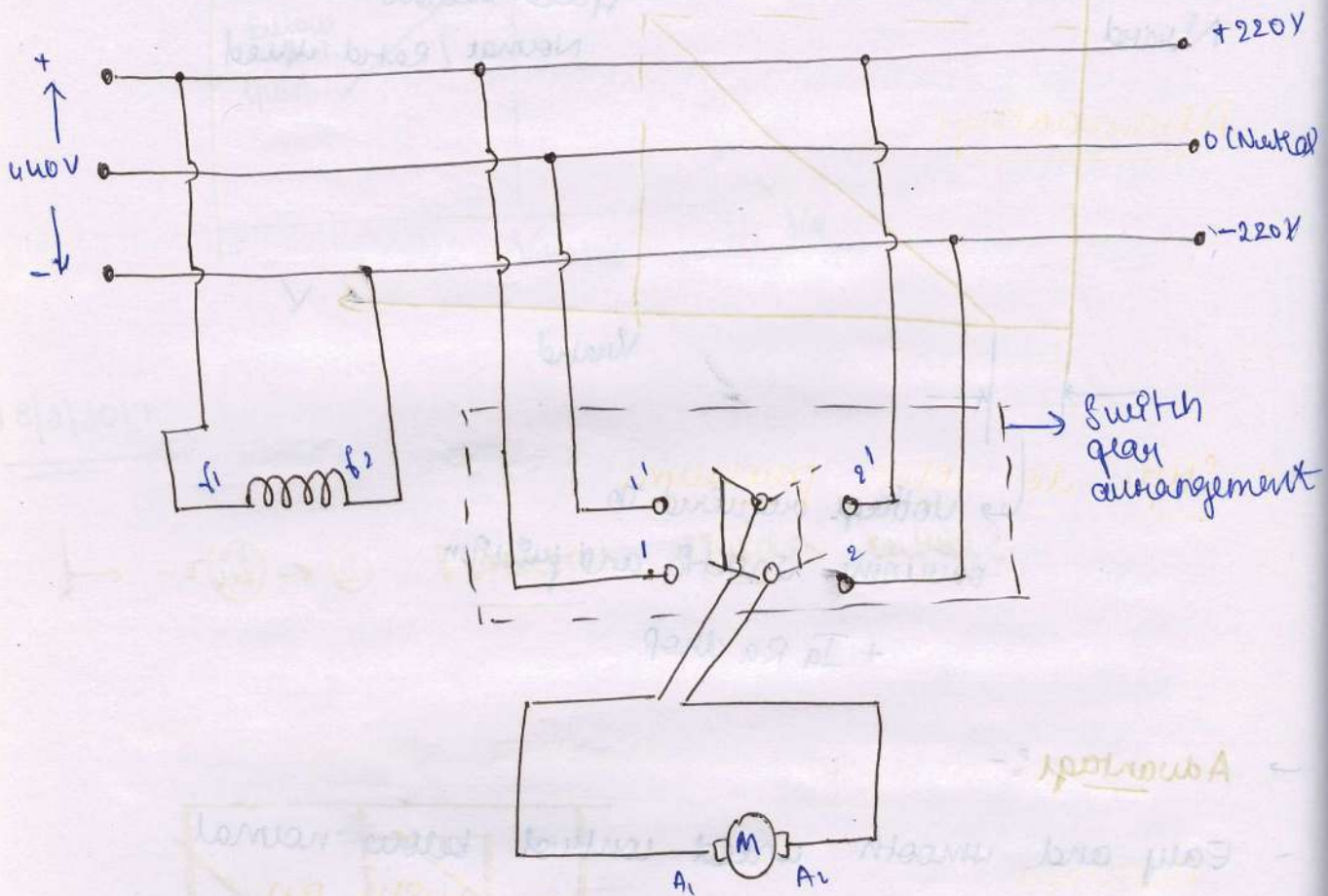
→ Advantages:-

- Easy and smooth speed control below normal speed is possible

⇒ Disadvantages:-

- As the entire I_a passes through the internal resistance there is high power loss
- Rheostat required is of large size.
- Speed control above the stand speed is not possible

iii) Supply / Applied voltage control method :-



Advantages

- wide range of speed control by suitable arrangement in the field or armature circuit

- Speed control in both directions can be achieved
- Uniform acceleration

Disadvantages

- This arrangement is expensive
- Overall efficiency is low

→ Steps to solve problems

1) $T_a \propto \Phi I_a$

2) $N \propto \frac{E_b}{\Phi}$

Problems

- Q1) A 230V, DC shunt motor with an $R_a = 0.04 \Omega$, is excited to give constant field current. The motor runs at 500 rpm at full load and takes armature current of 30 Amps. If an external resistance of $1.01 \Omega (R_x)$ is placed in the armature circuit, Find the speed of the motor at (i) full load torque (ii) 1.5 times full load torque.

Soln :-

Given :-

$$V = 230 \text{ V}$$

$$R_a = 0.4 \Omega$$

$$R_{ex} =$$

$$I_{a1} = 30 \text{ A}$$

$$R_x = 101 \Omega$$

$$N_1 = 500 \text{ rpm}$$

To find :-

$$N_2 = ?$$

1) (i) $\rightarrow$ Under full load torque condition -

$$\underline{T_{a1} = T_{a2}}$$

$$\text{Then } T_a \propto \phi I_a$$

[constant General torque condition]

$$T_a \propto I_a \quad [\text{shunt motor}]$$

$$\bullet \hookrightarrow T_{a1} \propto I_{a1}$$

$$\hookrightarrow T_{a2} \propto I_{a2}$$

$$I_{a1} = I_{a2} = 30 \text{ A}$$

[from above eqn]

$$N \propto \frac{E_b}{\phi}$$

[general eqn]

$$N \propto E_b$$

[$\phi = \text{constant}$]

$$N_1 \propto E_{b1} \rightarrow (1)$$

$$N_2 \propto E_{b2} \rightarrow (2)$$

Taking ratio of (1) and (2)

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

$$\rightarrow E_{b1} = V - I_{a1} R_a$$

$$\rightarrow E_{b2} = V - I_{a2} (R_a + R_x)$$

$$\rightarrow E_{b1} = 230 - 30 \times 0.4$$

$$= 218 \text{ V}$$

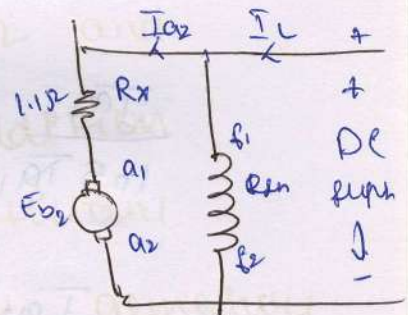
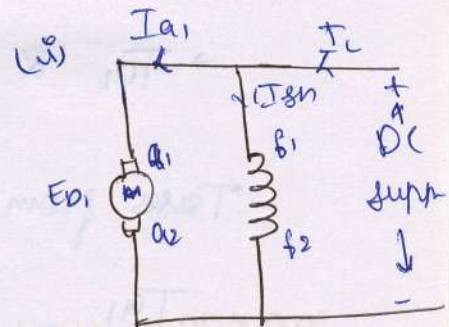
$$\rightarrow E_{b2} = 230 - 30 (0.4 + 1.1)$$

$$= 185 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

$$N_2 = \frac{1500 \times 185}{218}$$

$$N_2 = 1244 \text{ rpm}$$



g) ii) Under 105 times full load torque

$$T_{a2} = 105 T_{a1}$$

$$T_a \propto I_a$$

$$\rightarrow T_{a1} \propto I_{a1} \rightarrow (1)$$

$$\rightarrow T_{a2} \propto I_{a2} \rightarrow (2)$$

Take from (1) and (2)

$$\frac{T_{a1}}{T_{a2}} = \frac{I_{a1}}{I_{a2}}$$

$$\frac{T_{a1}}{105 T_{a1}} = \frac{I_{a1}}{I_{a2}}$$

$$I_{a2} = 105 I_{a1}$$

$$= 105 \times 30$$

$$\underline{I_{a2} = 45 \text{ Amps}}$$

$$\hookrightarrow E_{b1} = V - I_{a1} R_a$$

$$= 230 - (30 \times 0.04)$$

$$= 218$$

$$\hookrightarrow E_{b2} = V - I_{a2} (R_a + R_x)$$

$$= 230 - 45 (0.04 + 0.01)$$

$$= 162.5 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

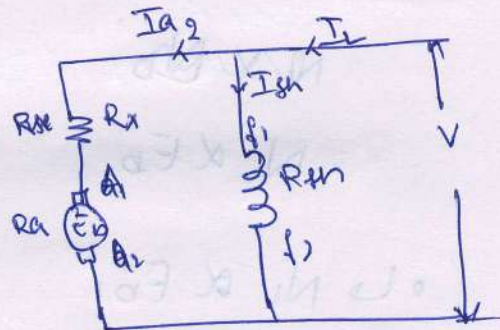
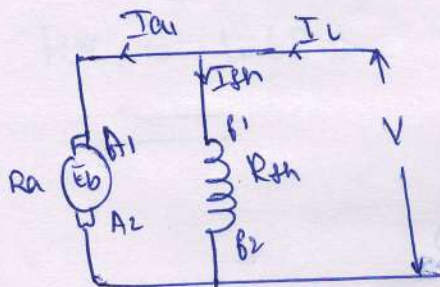
$$\frac{500}{N_2} = \frac{218}{16205}$$

$$N_2 = \underline{\underline{3720.70 \text{ rev}}}$$

$$N_2 = \underline{\underline{373 \text{ rpm}}}$$

Q2) A DC shunt motor runs at 1000 rpm on 200V supply, $R_a = 0.8 \Omega$ and the current taken is 40 Amps in addition to the field current. What resistance must be connected in series with armature to reduce the speed to 600 rpm, the armature torque remaining the same.

Soln



Given

$$V = 200 \text{ V}$$

$$R_a = 0.8 \Omega$$

$$I_{a1} = 40 \text{ A}$$

$$N_1 = 1000 \text{ rpm}$$

$$N_2 = 600 \text{ rpm}$$

$$\parallel T_{a1} = T_{a2}$$

To find :-

$$R_x = ?$$

⇒ General Torque eqn

$$T_a \propto \phi I_a$$

$$T_a \propto I_a$$

[DC shunt motor
 $\phi = \text{constant}$]

$$\therefore T_{a1} \propto I_{a1}$$

$$\therefore T_{a2} \propto I_{a2}$$

$$T_{a1} = T_{a2}$$

$$I_{a1} = I_{a2} = 40 \text{ Amps}$$

$$N \propto \frac{E_b}{\phi}$$

$\phi = \text{constant}$

$$N \propto E_b$$

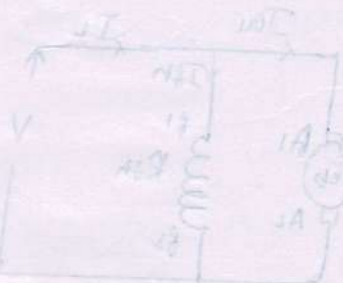
$$N \propto E_b$$

$$\therefore N_1 \propto E_{b1}$$

①

$$\therefore N_2 \propto E_{b2}$$

②



to (1) & (2) A 330V, 100W, 50Hz

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

$$\rightarrow E_{b1} = V - I_{a1} R_a$$

$$= 200 - (40 \times 0.8)$$

$$= 168V$$

$$\rightarrow E_{b2} = V - I_{a2}(R_a + R_x)$$

$$200 = 100.8$$

$$\hookrightarrow \frac{1000}{600} = \frac{168}{E_{b2}} \Rightarrow E_{b2} = 100.8V$$

$$100.8 = 200 - 40(0.8) - 40 \times R_x$$

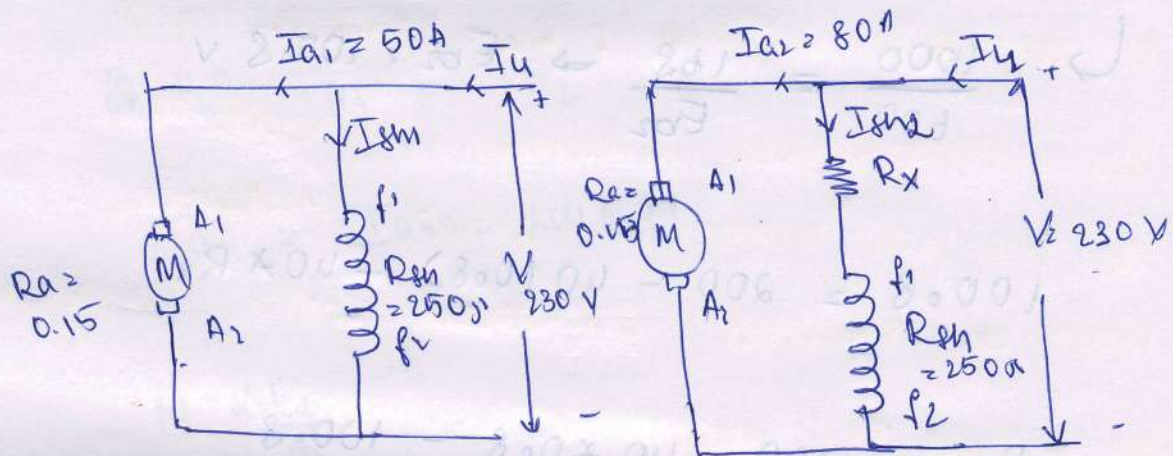
$$R_x = \frac{200 - 40 \times 0.8 - 100.8}{40}$$

$$R_x = 1.68 \Omega$$

==

Q3) A 230 volts, DC shunt motor runs at 800 rpm and takes armature current of 50 Amps. Find the resistance to be added in the field circuit to increase the speed from 800 rpm to 1000 rpm, at an armature current of 80 Amps. Assume flux proportion to field current-. $R_a = 0.15 \Omega$ and $R_{sh} = 250 \Omega$

Soln



Given

$$V = 230$$

$$R_a = 0.15 \Omega$$

$$R_{sh} = 250 \Omega$$

$$I_{a1} = 50A$$

$$I_{a2} = 80A$$

$$N_1 = 800 \text{ rpm}$$

$$N_2 = 1000 \text{ rpm}$$

$$\Phi \propto I_{sh} \propto I_f$$

To find:- $R_x = ?$

$$\rightarrow \cancel{I_a} \neq I_a$$

$$\rightarrow V$$

$$\rightarrow I_{sh1} = \frac{V}{R_{sh}}$$

$$I_{sh1} = \frac{250}{230}$$

$$I_{sh1} = \underline{0.92A}$$

$$\rightarrow E_{b1} = V - I_{a1} R_a$$

$$= 230 - 50 \times 0.15$$

$$E_{b1} = \underline{222.5V}$$

$$\rightarrow E_{b2} = V - I_{a2} R_a$$

$$= 230 - 80 \times 0.15$$

$$E_{b2} = \underline{218V}$$

Note :-

As load condition is not given,
torque equation cannot be used

$$\rightarrow N \propto \frac{E_b}{\Phi}$$

$$\left[\Phi \propto I_{sh} \propto I_f \right]$$

$$\rightarrow N \propto \frac{E_b}{I_{sh}}$$

$$\rightarrow N_1 \propto \frac{E_{b1}}{I_{sh1}} \quad \text{--- (1)}$$

$$\rightarrow N_2 \propto \frac{E_{b2}}{I_{sh2}} \quad \text{--- (2)}$$

$$(1) \div (2)$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{I_{sh1}} \times \frac{I_{sh2}}{E_{b2}}$$

$$\frac{800}{1000} = \frac{220.5}{0.92} \times \frac{I_{sh2}}{218}$$

$$I_{sh2} = 0.72 \text{ Amps}$$

Ohm's law

$$\rightarrow V = I_{sh2} (R_x + R_{sh})$$

$$\frac{E_b}{I_{sh}}$$

$$230 = 0.72 R_x + 0.72 \times 250$$

$$R_x =$$

$$R_x = \frac{230 - 0.72 \times 250}{0.72}$$

$$R_x = 69.44 \, \Omega$$

⇒ Speed control of DC ~~series~~ motor :-

↳ Field control

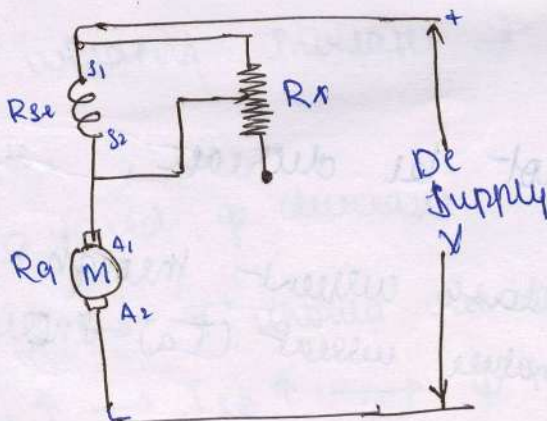
Note :-

The flux produced by a winding depends on the mmf (magnetomotive force) which is a product of current and no. of turns of the winding

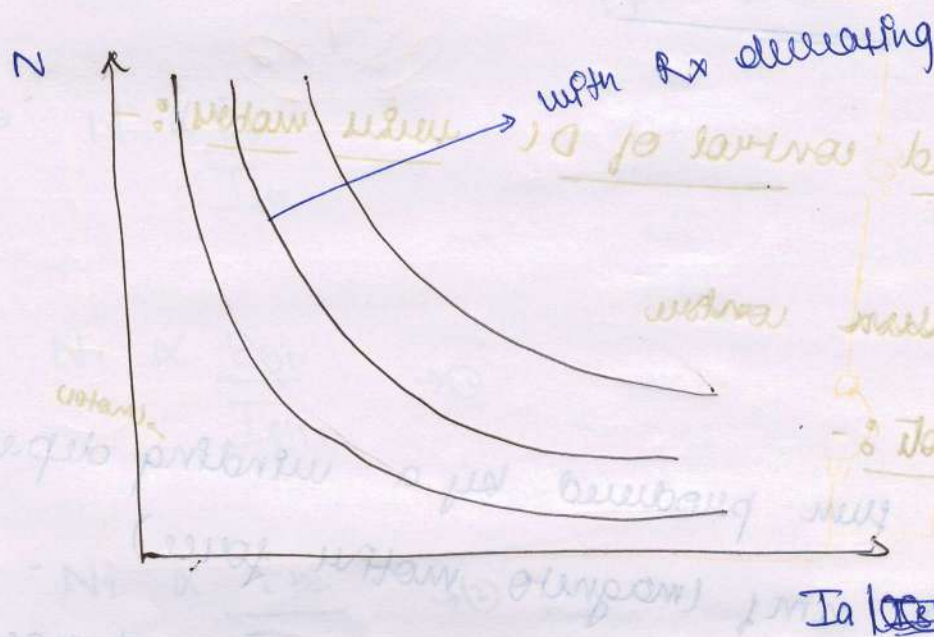
$$\text{mmf} = NI \quad \text{Amp Turns}$$

↳ Field control method

↳ (a) Field diverter method :-



~~A~~ $\rightarrow N \quad V_s \quad I_a \text{ (or) } I_{se} \text{ (or) } I_L$



$$N \propto \frac{E_b}{\Phi}$$

as $\Phi \uparrow$

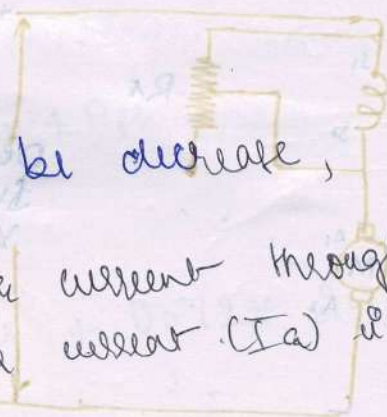
$\Rightarrow N \downarrow$

$$; I_a \uparrow ;$$

$$I_{se} \downarrow \rightarrow \Phi \downarrow \rightarrow N \uparrow$$

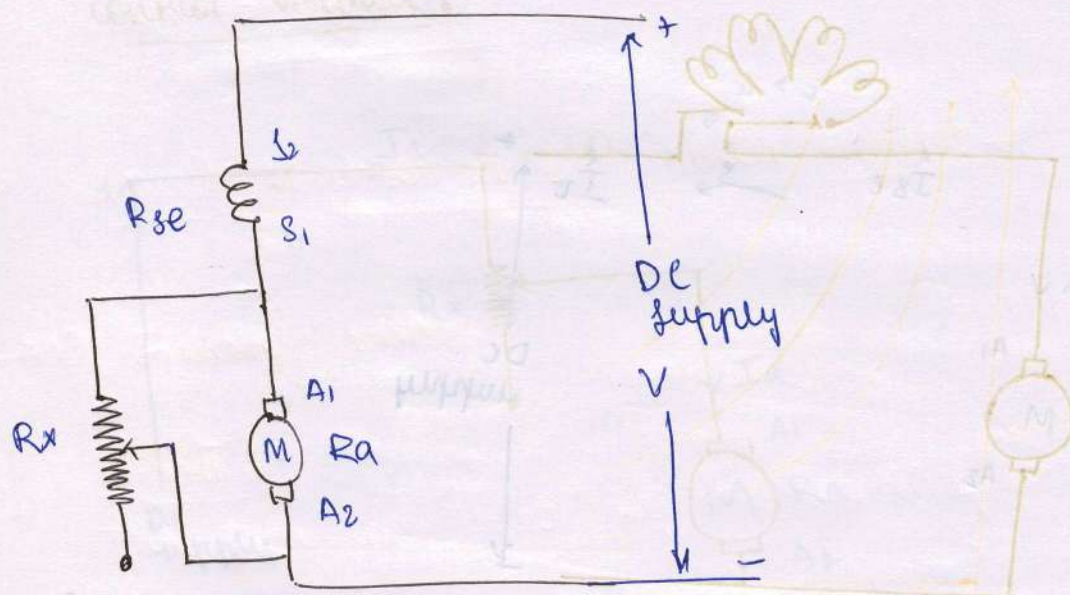
- Speed control above rated speed cannot be done.

- As flux cannot be decrease,



As Resistance R_x increase current through Rheostat decreases and the armature current (I_a) increases

b) Armature diverter method:-



As $\rightarrow$ Load $\uparrow \rightarrow I_a \uparrow \rightarrow \Phi \uparrow \rightarrow N \downarrow$

$\rightarrow$ when the armature current is diverted through R_x thus reducing it.

Now $\omega \propto \tau$

$$T_a \propto \Phi I_a$$

and since the load torque remains constant, the motor draws more current from the supply. Thus the current ~~to~~ through the series field winding increases $\rightarrow$ so does the flux increase.

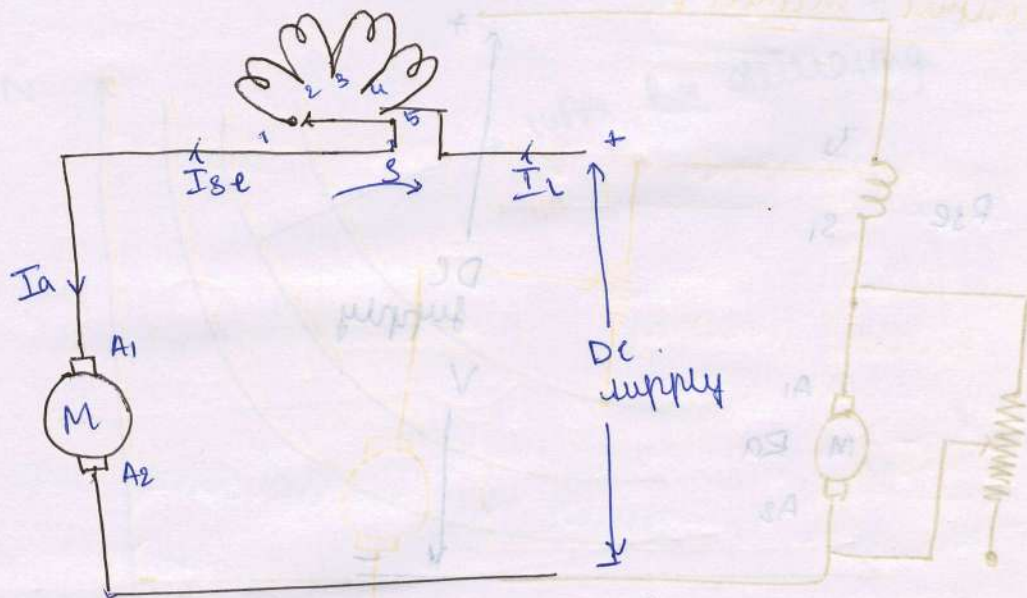
Thus speed Φ decreases

$\rightarrow I_a \downarrow \rightarrow T$ should remain constant $\rightarrow$

$\left(I_f \uparrow \rightarrow I_{se} \uparrow \rightarrow \Phi \uparrow \rightarrow N \downarrow \right)$

$\rightarrow T$ increases due to diverter.

c) Tapped field method :-



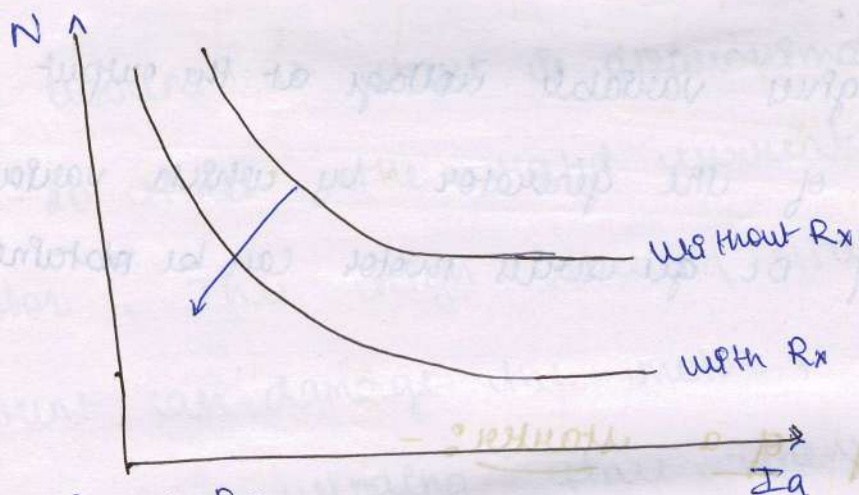
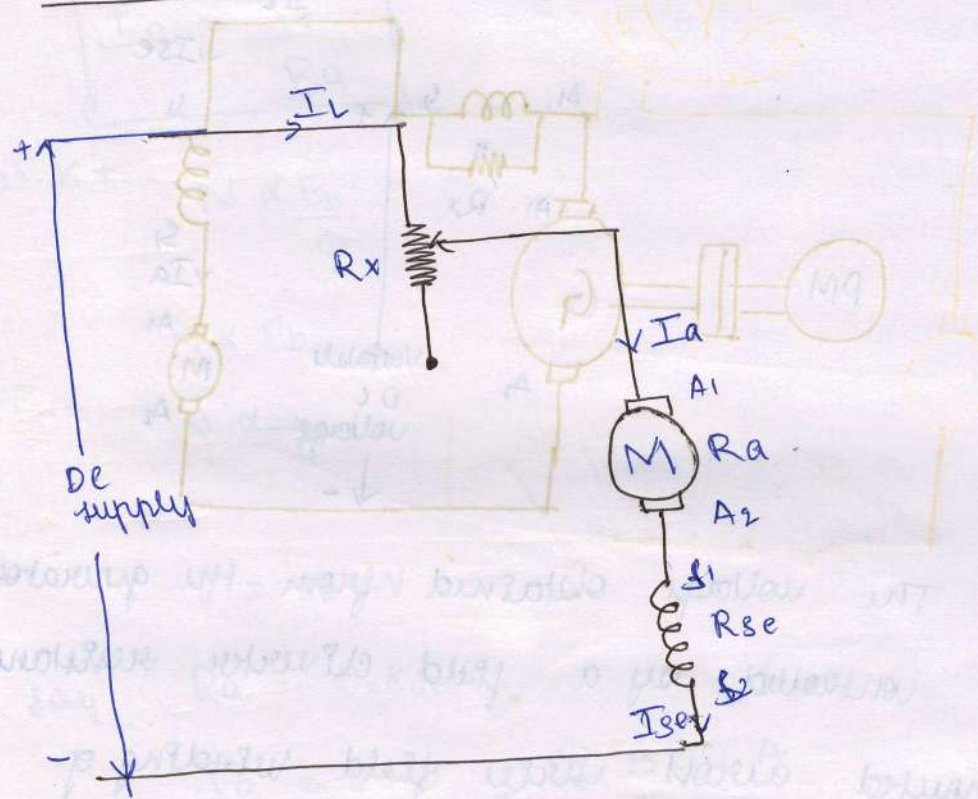
- In this method, the change in speed is achieved by changing the no. of turns of the field winding.
- The field winding is provided with no. of taps and a selector switch 'S' to select the no. of turns (taps) as per requirement.

When S is in position '1' → entire winding is included → no. of turns is more → flux is high → speed decreases

$S \rightarrow 1 \rightarrow \text{No. of turns} \uparrow \rightarrow \phi \uparrow \rightarrow N \downarrow$
 $S \rightarrow 2 \rightarrow \text{No. of turns} \downarrow \rightarrow \phi \downarrow \rightarrow N \uparrow$
 $S \rightarrow 3 \rightarrow \text{No. of turns} \downarrow \downarrow \rightarrow \phi \downarrow \downarrow \rightarrow N \uparrow \uparrow$

Q2) Rheostatic control (or) Armature voltage control method :-

control method :-



(i) without R_x
 $N \propto \frac{1}{\Phi}$

$$\Phi \propto I_{se}$$

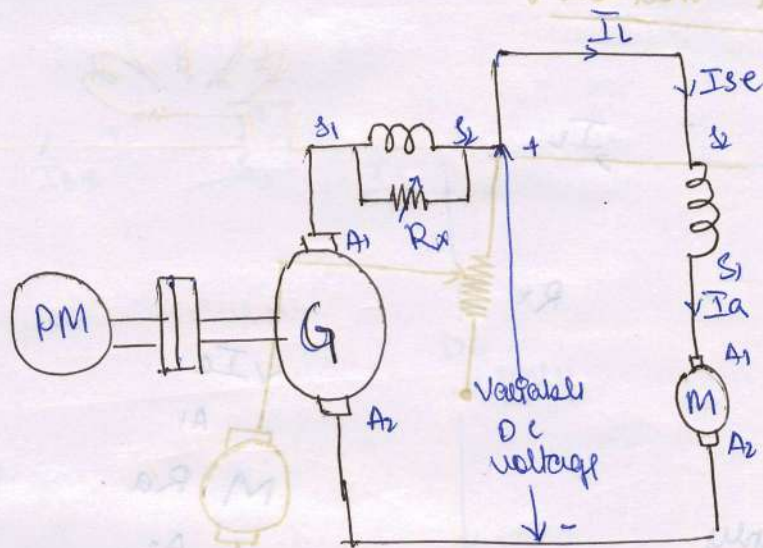
(ii) with R_x
 $N \propto V$

$N \downarrow \Rightarrow V \downarrow$

$$[N \propto E_b / \Phi \Rightarrow N \propto \Phi]$$

[$V \rightarrow \downarrow \rightarrow$ due to voltage drop]
 as R_x is added $I_a \downarrow$
 $\therefore V \uparrow$

→ 3) Applied voltage control method:-



→ The voltage obtained from the generator is controlled by a field diverter resistance connected across series field winding of generator.

- This gives variable voltage at the output terminals of the generator by which various speeds of DC series motor can be obtained.

⇒ Necessity of a starter:-

Consider a DC shunt motor -

$$V = E_b + I_a R_a$$

At start, $N = 0$

then $E_b = 0$



$$V = 0 + I_a R_a$$

$$V = I_a R_a$$

$$I_a = \frac{V}{R_a}$$

$$E_b = \frac{N \times \Phi}{10^8} \times \omega$$

$$N \propto \frac{E_b}{\Phi}$$

$$N \propto \frac{1}{\Phi}$$

Eg:-

$$V = 220 \text{ V}$$

say $R_a = 0.8 \Omega$

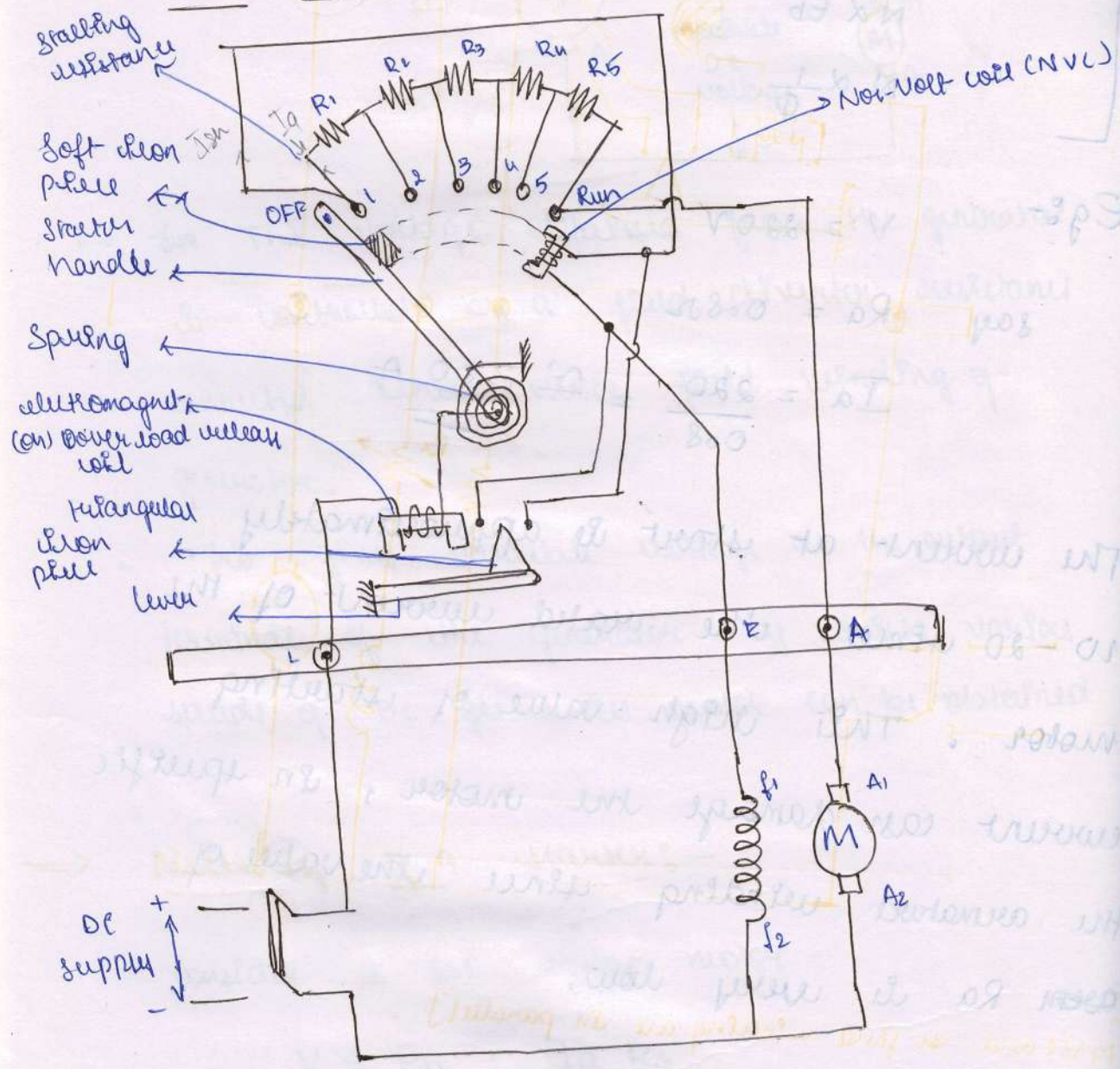
$$I_a = \frac{220}{0.8} = \underline{\underline{\approx 300 \text{ A}}}$$

The current at start is approximately 10-20 times the rated current of the motor. This high value of starting current can damage the motor, in specific the armature winding since the value of R_a is very low.

⇒ Starter :-

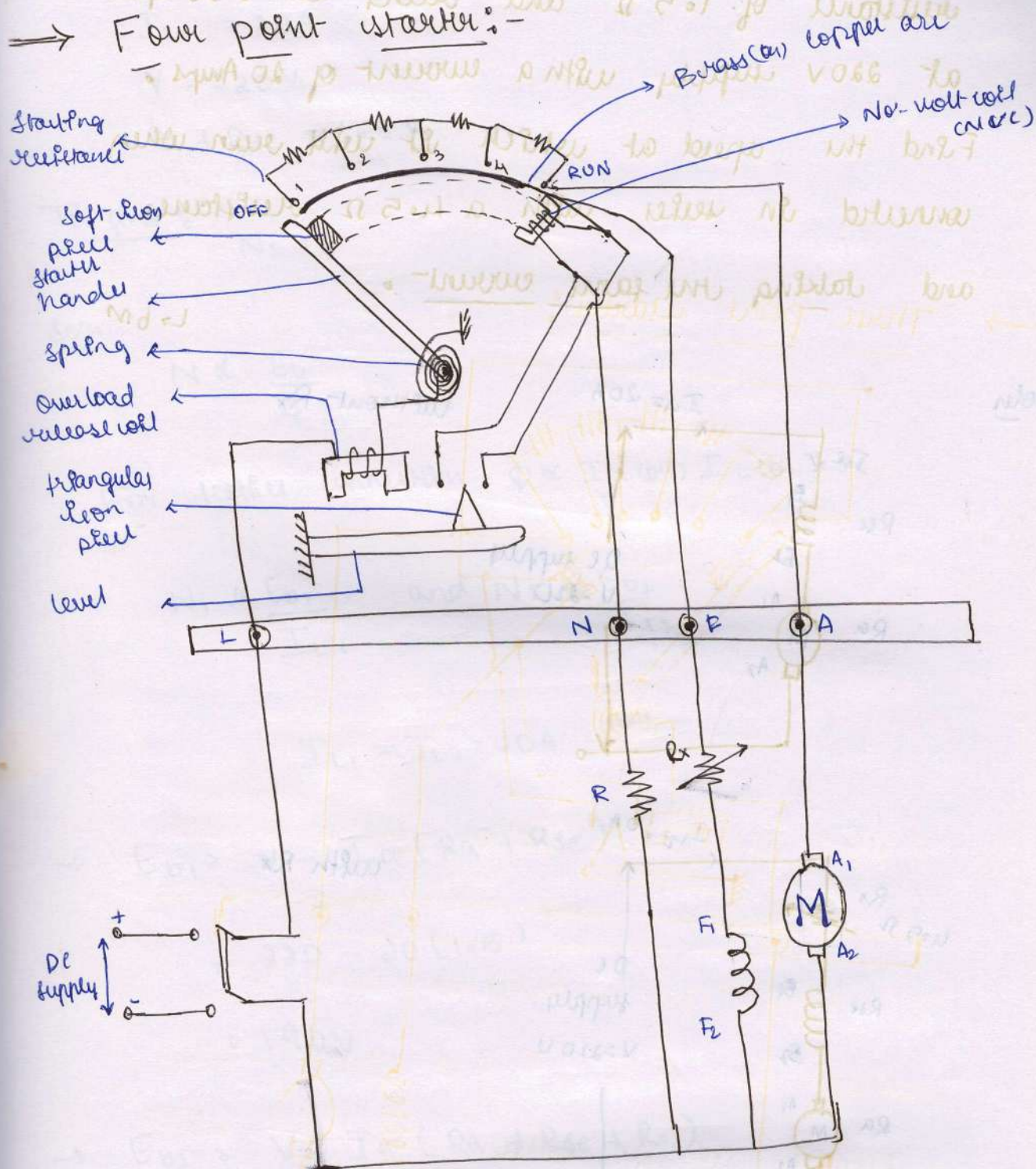
A starter is a safety device which is used to safely start a motor by limiting high starting current.

⇒ Three point starter :-



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Four point starter:-

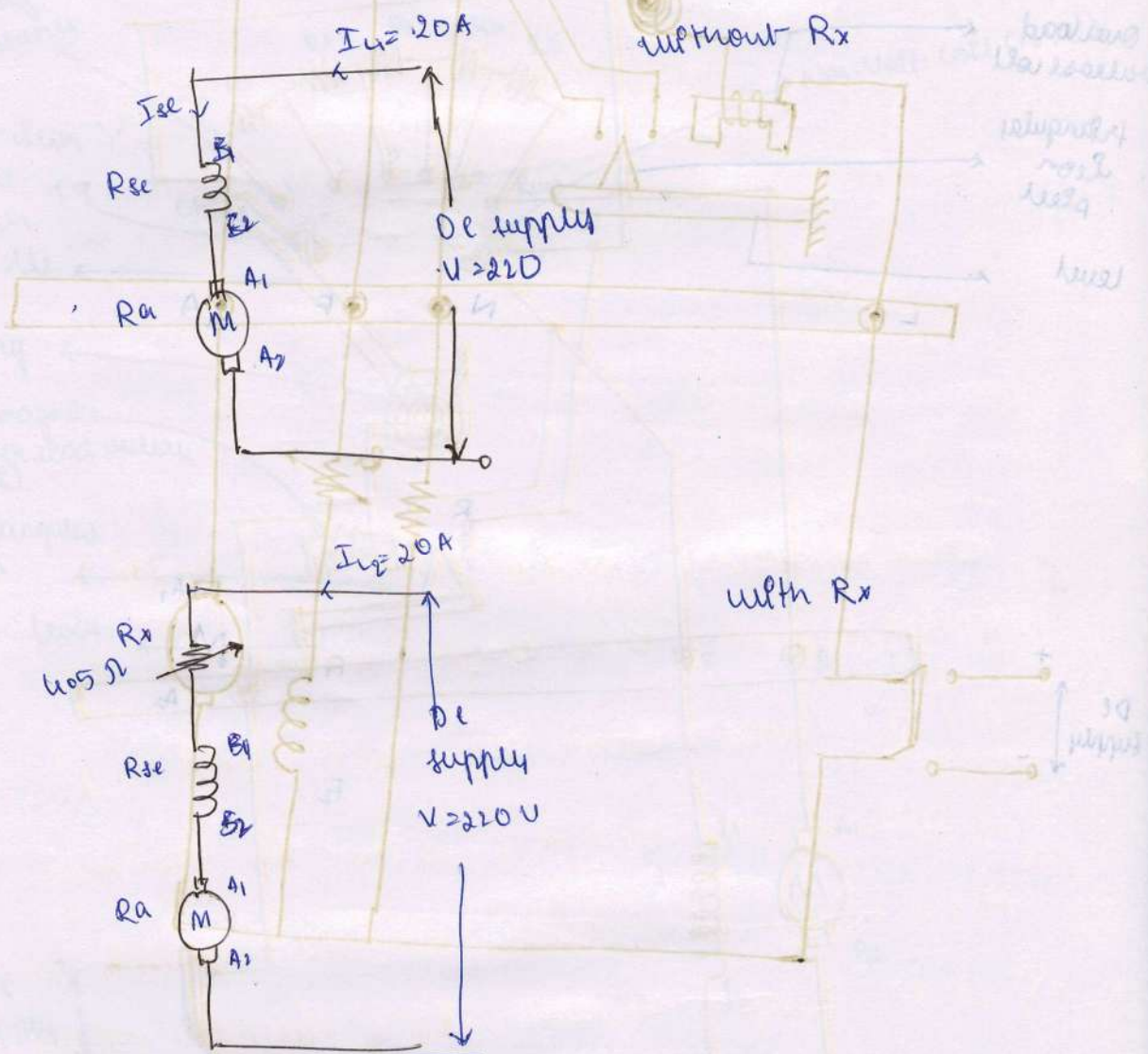


[NVC and field winding are in parallel]

Q1) A DC shunt motor has a total resistance of 1.05Ω and runs at 750 rpm at 220 V supply with a current of 20 Amps .

Find the speed at which it will run when connected in series with a 4.05Ω resistance and taking the same current.

Soln



Given

$$N_1 = 750 \text{ rpm}$$

$$I_{L1} = 20 \text{ A}$$

$$V = 220 \text{ V}$$

$$R_a + R_{se} = 1.05 \Omega$$

To find :-

$$N_2 = ?$$

Soln

$$N \propto \frac{E_b}{\phi}$$

For series connection $\phi \propto I_a(\omega) I_L(\omega) I_{se}$

$$N_1 \propto \frac{E_{b1}}{I_{L1}} \quad \text{and} \quad N_2 \propto \frac{E_{b2}}{I_{L2}}$$

$$I_{L1} = I_{L2} = 20 \text{ A}$$

$$\rightarrow E_{b1} = V - I_a(R_a + R_{se})$$

$$= 220 - 20(1.05)$$

$$= 199 \text{ V}$$

$$\rightarrow E_{b2} = V - I_a(R_a + R_{se} + R_x)$$

$$= 220 - 20(1.05 + 1.05)$$

$$= 100 \text{ V}$$

$$N_1 \propto \frac{E_{b1}}{I_{L1}}$$

$$N_2 \propto \frac{E_{b2}}{I_{L2}}$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{L2}}{I_{L1}}$$

$$\therefore N_2 = \frac{750 \times 990}{100} \times \frac{N_1 \times E_{b2}}{E_{b1}}$$

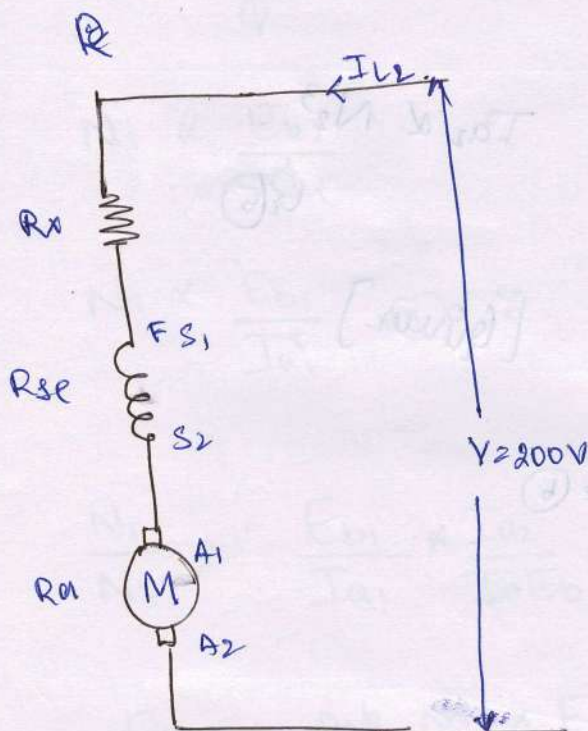
$$N_2 = \frac{750 \times 100}{100}$$

$$= 394.73$$

$$\underline{N_2 \approx 395 \text{ rpm}}$$

Q2) A 200 V, DC series motor drives a load at a certain speed and takes a current of 30 Amps. The resistance b/w its terminals is 1.5Ω . Find the extra resistance to be added in series with motor circuit. To reduce speed to 60% of its original value. Assume torque produced is proportional to cube of speed

Soln



Given

$$I_{L2} = I_{a1} = 30A$$

$$R_a + R_{se} = 105\Omega$$

$$N_2 = 60\% N_1$$

$$\Rightarrow N_2 = 0.6 N_1$$

$$T \propto N^3$$

reduced by 60%
to 60% of N_1

To find:-

$$R_x = ?$$

Soln

$$T_a \propto \Phi I_a$$

$$T_a \propto I_a^2$$

for DC shunt motor

$$T_{a1} \propto I_{a1}^2$$

①

$$T_{a2} \propto I_{a2}^2$$

②

$$\frac{I_{a1}}{I_{a2}} = \frac{I_{a1}^2}{I_{a2}^2} \rightarrow (3)$$

$$I_{a1} \propto N_1^3 \quad \text{and} \quad I_{a2} \propto N_2^3$$

↳ (4)

[given]

$$\frac{I_{a1}}{I_{a2}} = \frac{N_1^3}{N_2^3} \rightarrow (5)$$

equation (3) and (5)

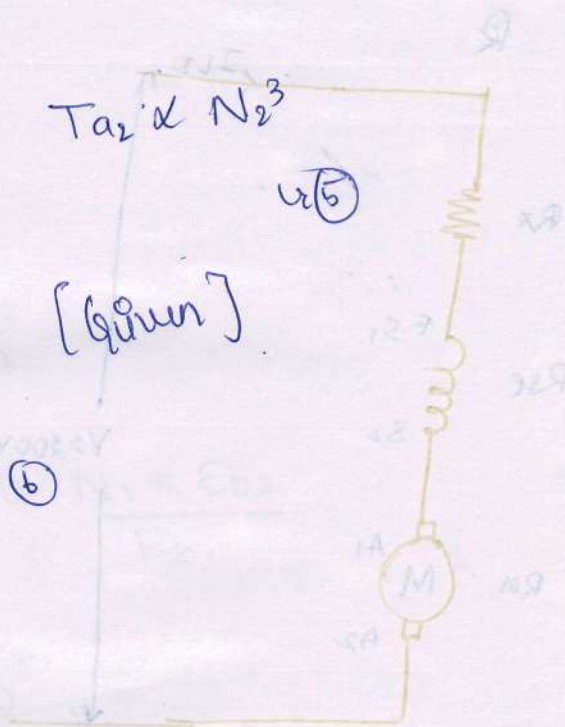
$$\frac{I_{a1}^2}{I_{a2}^2} = \frac{N_1^3}{N_2^3}$$

$$\frac{I_{a1}^2}{I_{a2}^2} = \frac{(N_1)^3}{(0.6 N_1)^3}$$

$$\frac{(30)^2}{I_{a2}^2} = \frac{1}{0.216}$$

$$I_{a2}^2 = 19404$$

$$I_{a2} = \underline{\underline{13.94 \text{ Amp}}}$$



$$N \propto \frac{E_b}{\Phi}$$

$$N_1 \propto \frac{E_{b1}}{\Phi_1}$$

$$N_2 \propto \frac{E_{b2}}{\Phi_2}$$

$$N_1 \propto \frac{E_{b1}}{I_{a1}}$$

$$N_2 \propto \frac{E_{b2}}{I_{a2}} \quad (8)$$

$$\frac{N_1}{N_2} = \frac{E_{b1} \times I_{a2}}{I_{a1} \times E_{b2}}$$

$$E_{b2} = \frac{0.6 N_1 \times E_{b1} \times I_{a2}}{0.6 N_2 \times I_{a1}}$$

$$\rightarrow E_{b1} = V - I_{a1}(R_a + R_{se})$$

$$= 200 - 30(105)$$

$$= 155$$

$$\rightarrow E_{b2} = \frac{155 \times 13.94 \times 0.6}{0.6 \times 30}$$

$$= 43.021 \text{ V}$$

$$\rightarrow E_b = V - I_{a2}(R_a + R_{se} + R_x)$$

$$43.021 = 200 - 13.94(105 + R_x)$$

$$R_x = \frac{200 - 43.021 - 13.94 \times 105}{13.94} = 9.74 \Omega$$

→ Loss and efficiency :-

Losses present in a D.C motor are

1) Copper losses (or) Variable losses - P_{cu}

2) ~~Excitation loss~~

3) Mechanical losses

2) Iron losses

↳ 1) Copper loss :-

Variable losses

- It is also called $I^2 R$ losses where

current I is the variable parameter

and generally R remains constant.

(It takes place in winding $\rightarrow$ Cu material)

↳ 2) Iron losses :-

Constant losses (or) Magnetic losses

Denoted as by ' P_i '

(It takes place in core - $\rightarrow$ magnetic material)

↳ (i) Hysteresis loss :-

Rapid cycle of magnetisation and demagnetisation

To reduce hysteresis loss used a very good magnetic material with low

→ ii) Eddy current losses:-

The current flow in the unwanted portions of the core is called eddy currents.

To reduce eddy current loss - laminated construction of core is used.

→ 3) Mechanical losses:-

These losses are also called as windage

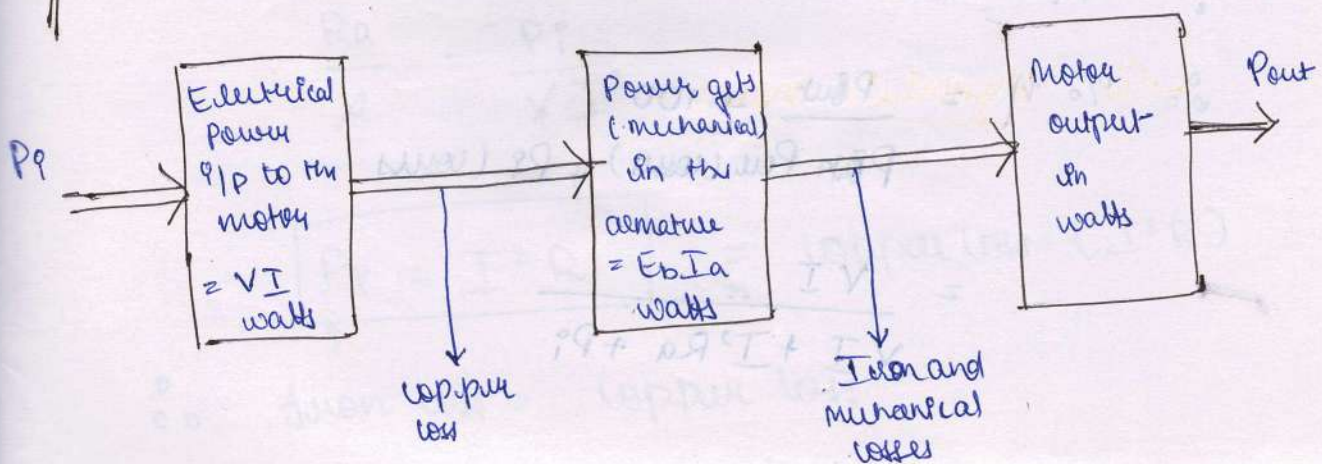
(or) frictional losses

constant loss (or) Magnetic losses + mechanical loss → stray losses
(constant) (constant) (constant)

Total losses :-

Total loss = constant losses + variable losses.

→ Power flow diagram :- for DC motor



→ Efficiency :-

$$\% \text{ Efficiency} = \frac{P_{out}}{P_{in}} \times 100$$

for motor

$$\% \eta_m = \frac{P_{in} - \text{total loss}}{P_{in}} \times 100$$

for generator

$$\% \eta_g = \frac{P_{out}}{P_{out} + \text{losses}} \times 100$$

motor
electrical
i/p power

generator
electrical
o/p power

→ Condition for maximum efficiency :-

mechanical is variable
or variable whereas
electrical load is constant
or constant % using

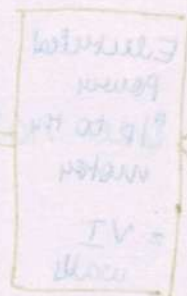
consider a DC generator

$$\text{Let } P_{out} = VI$$

$$P_{cu} = \text{variable loss} = I_a^2 R_a \text{ (or) } I^2 R_a$$

$$P_i = \text{constant loss}$$

$$\begin{aligned} \% \eta &= \frac{P_{out}}{P_{out} + P_{cu} (\text{loss}) + P_i (\text{loss})} \times 100 \\ &= \frac{VI}{VI + I^2 R_a + P_i} \times 100 \end{aligned}$$



∴ both numerator and denominator by $V I$

$$\% \eta = \frac{V I}{\frac{V I}{V I} + \frac{I^2 R_a}{V I} + \frac{P_i}{V I}}$$

$$\% \eta = \frac{1}{\left[1 + \frac{I R_a}{V} + \frac{P_i}{V I} \right]} \times 100$$

For efficiency to be maximum, the denominator should be minimum according to maxima minima theorem

$$\therefore \frac{d}{dI} \left[1 + \frac{I R_a}{V} + \frac{P_i}{V I} \right] = 0$$

$$0 + \frac{R_a}{V} + - \frac{P_i}{V I^2} = 0$$

$$\frac{R_a}{V} = \frac{P_i}{V I^2}$$

$$\boxed{P_i = I^2 R} = \text{copper loss } (I^2 R)$$

∴ Iron loss = copper loss

Q1) A 6 pole 500 volts, wave connected

DC shunt motor has 1200 armature conductors and flux per pole is 20 mWb . The armature and field resistances are 0.5Ω and 250Ω respectively. What will be the speed and torque developed by the motor when it draws a current of 20 Amps from the supply? If the magnetic and mechanical losses are 900 W . Find

(i) Useful torque

(ii) Efficiency at this load

Soln

Given

$$P = 6$$

$$V = 500 \text{ V}$$

$$\text{Wave wound } A = 2$$

$$Z = 1200$$

$$\Phi = 20 \times 10^{-3} \text{ Wb}$$

$$R_a = 0.5 \Omega$$

$$R_{sh} = 250 \Omega$$

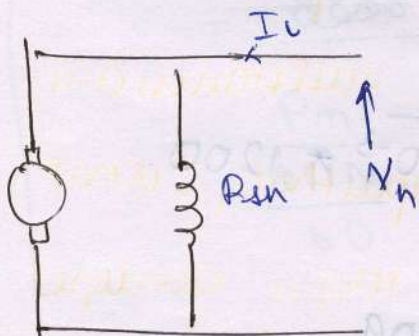
$$I_L = 20 \text{ A}$$

To find:-

1) $I_{sh} = ?$

2) $\gamma. \eta$

3) N



$$I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{500}{250} = 2A$$

$$I_L = I_a + I_{sh}$$

$$I_a = I_L - I_{sh}$$

$$= 20 - 2$$

$$= 18A$$

$$E_b = \frac{P \Phi N Z}{60A}$$

$$E_b = V - I_a R_a$$

$$= 500 - 18 \times 0.05$$

$$= 491V$$

$$E_b = \frac{P \phi N Z}{60 A}$$

$$N = \frac{E_b 60 A}{P \phi Z}$$

$$= \frac{491 \times 60 \times 2}{6 \times 20 \times 10^{-3} \times 1200}$$

$$= 409$$

$$N = \approx 409 \text{ rpm}$$

$$T = \frac{P \phi Z I_a}{2 \pi A}$$

$$= \frac{6 \times 20 \times 10^{-3} \times 1200 \times 18}{2 \pi \times 2}$$

$$T = 206.26 \text{ Nm}$$

Power developed in the armature

$$P_m = E_b \times I_a$$

$$= 491 \times 18$$

$$P_m = \underline{\underline{8838 \text{ watts}}}$$

$$P_m = T_{\text{em}} \times \omega = T_a \times \omega$$

← Power developed in armature

$$T_{\text{em}} = \frac{P_m}{\omega}$$

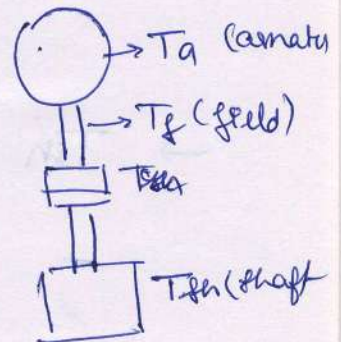
$$= \frac{8838}{60}$$

$$= \frac{P_m}{\frac{2\pi N}{60}}$$

$$= \frac{P_m \times 60}{2\pi N}$$

$$= \frac{8838 \times 60}{2 \times \pi \times 406}$$

$$T_{\text{em}} T_a = 206.26 \text{ Nm}$$



same

$$P_{\text{em}} = V I_a$$

$$= 500 \times 18$$

$$= 9000 \text{ watts}$$

$$\rightarrow P_{\text{em}} = V I_L$$

$$= 500 \times I_L$$

$$= 500 \times 20$$

$$P_{\text{em}} = 10,000 \text{ watts}$$

[electrical parameter
i/p in motor]

$$P_{out} = P_m - \text{Mechanical loss}$$

$$= 8838 - 900$$

$$P_{out} = \underline{7938 \text{ watts}}$$

$$T_{sh} = \frac{P_{out}}{\omega}$$

$$= \frac{7938 \times 60}{2 \times \pi \times 409}$$

$$T_{sh} = \underline{185.33 \text{ N}}$$

% due
T_{sh} will
be less
than T_a

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{7938}{10000} \times 100$$

$$\% \eta = \underline{79.38\%}$$

MODULE-2

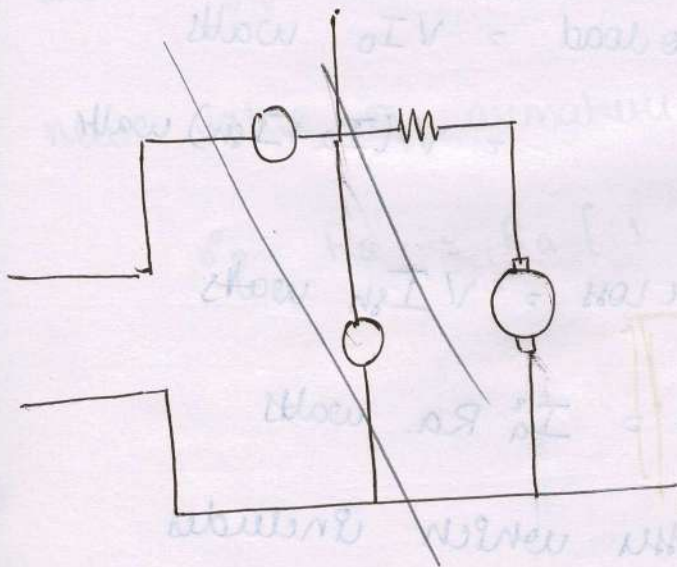
TESTING OF DC MOTORS

⇒ Sub branches are -

↳ (i) Direct testing:-

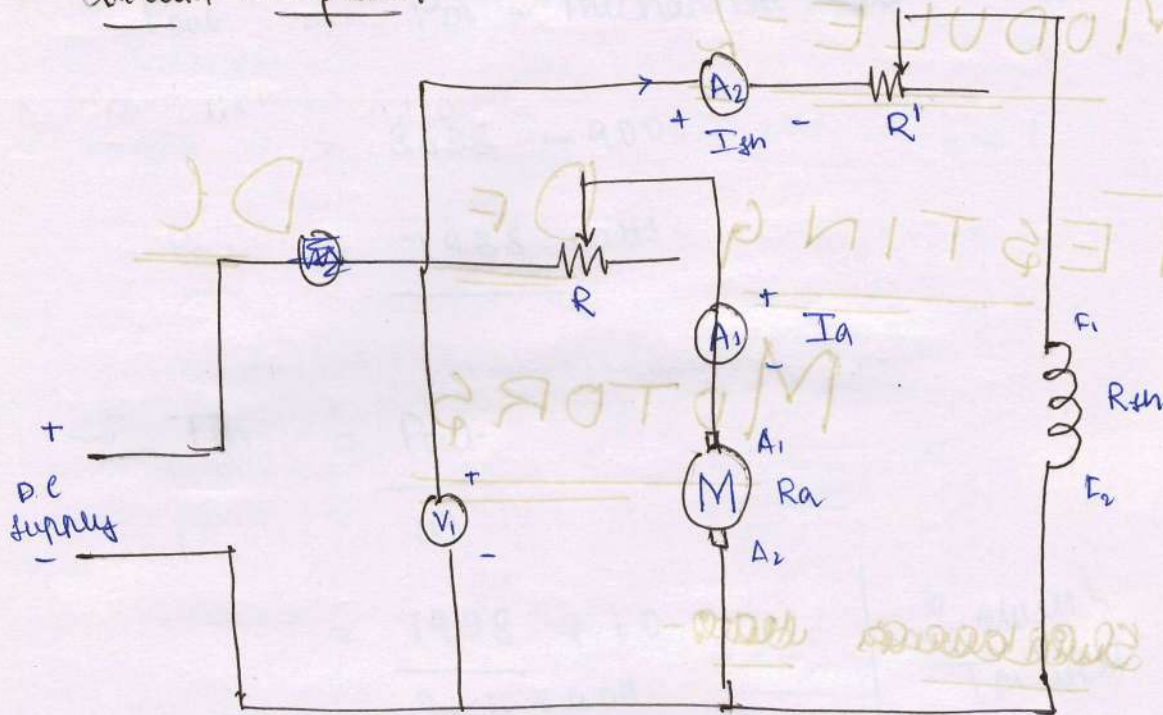
↳ (ii) Indirect testing:-

⇒ Swinburne's testing:- [Indirect method of testing]



→ Swinburne's M.M. -

circuit diagram:-



Tabular column:-

Sd. No.	V (volts)	I_a (Amps)	I_{sh} (Amps)	$I_o = I_a + I_{sh}$
---------	-----------	--------------	-----------------	----------------------

Calculation:-

If V is the supply voltage then

then

→ Power a/p on no load = $V I_o$ watts

$$= V (I_a + I_{sh}) \text{ watts}$$

∴

→ shunt field copper loss = $V I_{sh}$ watts

→ Armature copper loss = $I_a^2 R_a$ watts

Thus the stray losses which includes iron, friction and windage losses.

Stray $\rightarrow$ Iron loss + Mechanical loss

$\rightarrow$ Iron loss + (^{friction}friction + windage) loss

can be obtained

$$\text{Stray loss} = \text{No load } I/P - \text{field cu loss} - \text{Armature cu loss}$$

$$\therefore \text{Stray loss} = V I_0 - V I_{sh} - I_a^2 R_a = W_s$$

When current flows through the field and armature windings there will be copper losses ($I^2 R$) in the respective winding which increases the temperature, in turn it affects their resistances

Thus new value of shunt field resistance = R'_{sh}

and $\therefore R'_{sh} = R_{sh} [1 + \alpha_0]$ and

new value of armature resistance = R'_a

$$\therefore R'_a = R_a [1 + \alpha]$$

R'_a and R'_{sh} are temperature dependent

→ To calculate efficiency of the motor

at $1/4^{\text{th}}$ load:-

Let I_{FL} = full load current of the motor

W_F = Field copper loss

W_s = stray loss

Load current at $1/4^{\text{th}}$ load

$$= I_{FL} \times \frac{1}{4} = \frac{I_{FL}}{4}$$

Motor input at $1/4^{\text{th}}$ load

$$= V \times \frac{I_{FL}}{4} = \frac{V I_{FL}}{4}$$

Armature current at $1/4^{\text{th}}$ load

$$= I_a' = \frac{I_{FL}}{4}$$

$$I_a' = \frac{I_{FL}}{4} - I_{sc}$$

Armature copper loss at $1/4^{\text{th}}$ load = $(I_a')^2 R_a$

$$= (I_a')^2 R_a$$

$$= [I_a']^2 R_a$$

$$= \left(\frac{I_a}{4} \right)^2 R_a$$

$$= \left[\frac{I_{FL}}{4} - I_{sh} \right]^2 R_a$$

Motor output at $1/4$ th load = $\left(\frac{V I_{FL}}{4} \right) - \text{P-loss}$

$$= \left[\frac{V I_{FL}}{4} \right] - \left[\left(\frac{I_{FL}}{4} - I_{sh} \right)^2 R_a + W_F + W_s \right]$$

% Efficiency = $\frac{\text{Motor output}}{\text{Motor input}}$

$$= \left[\frac{V I_{FL}}{4} - \left(\frac{I_{FL}}{4} - I_{sh} \right)^2 R_a - W_F - W_s \right]$$

$$= \frac{\text{Input} - \text{loss}}{\text{Input}} \times 100$$

$$\% \eta_{1/4} = \frac{\frac{V I_{FL}}{4} - \left[\left(\frac{I_{FL}}{4} - I_{sh} \right)^2 R_a + W_F + W_s \right]}{\frac{V I_{FL}}{4}} \times 100$$

Q1) A 500 volts DC shunt motor takes 4 Amps on no load. The armature resistance including that of the brushes is 0.02Ω and the field current is 1 Amp. Estimate the output and efficiency when the o/p current is

(i) 20 Amps

(ii) 100 Amps

Soln Given:-
 $V = 500 \text{ V}$

$$I_o = 4 \text{ Amps}$$

$$R_a = 0.02 \Omega$$

$$I_{fh} = 1 \text{ Amps}$$

To find:- o/p efficiency

$$\text{at } I_{L1} = 20 \text{ A, } I_{L2} = 100 \text{ A}$$

Soln

$$\rightarrow I_{ao} = I_o - I_{fh}$$

$$I_{ao} = 3 \text{ Amps}$$

$\rightarrow$ No load armature copper loss = ?

$$= I_{ao}^2 R_a = (3)^2 (0.02) = \underline{1.8 \text{ watts}}$$

$$\rightarrow \text{No load i/p} = V I_o$$

$$500 \times 4 = \underline{2000 \text{ watts}}$$

$$\rightarrow \text{constant loss} = V I_a - I_a^2 R_a = 1000$$

$$= 2000 - 108$$

$$= \underline{1998.2 \text{ watts}}$$

→ when $I_L = 20 \text{ Amps}$

$$I_a = I_L - I_m$$

$$= 20 - 1 = \underline{19 \text{ Amps}}$$

$$\text{New value of } W_{\text{loss}} = I_a^2 R_a = 19^2 \times 0.2$$

$$= \underline{72.2 \text{ watts}}$$

$$\text{New load P/P} = V I_L$$

$$= 500 \times 20$$

$$= \underline{10000 \text{ watts}}$$

$$\% \eta = \frac{P/P - W_{\text{loss}}}{P/P} \times 100$$

$$= \frac{10000 - [72.2 + 1998.2]}{10000} \times 100$$

$$= \underline{79.29 \%}$$

→ when $I_L = 100 \text{ Amps}$

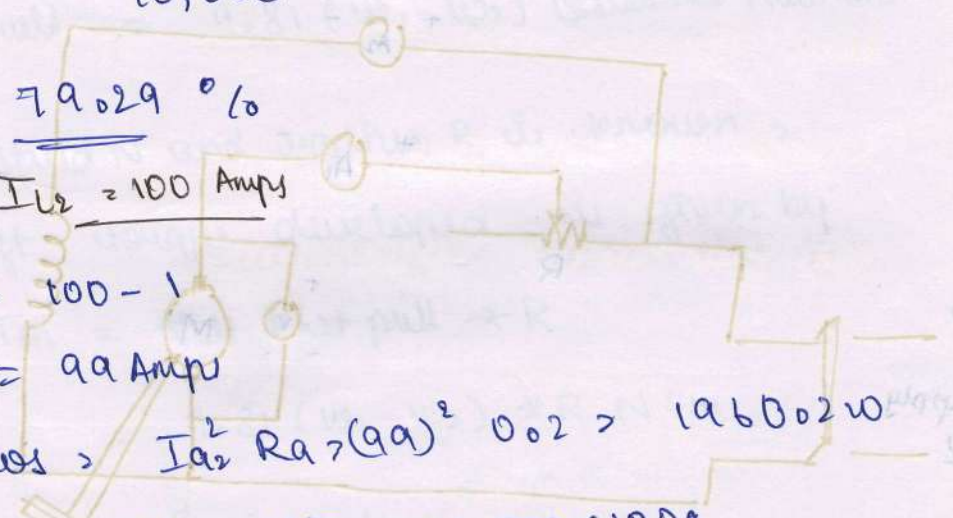
$$I_a = 100 - 1$$

$$= \underline{99 \text{ Amps}}$$

$$\text{New } W_{\text{loss}} = I_a^2 R_a = (99)^2 \times 0.2 = \underline{1960.2 \text{ W}}$$

$$\text{New load P/P} = V I_L = 500 \times 100$$

$$= \underline{50000}$$



$$\% \eta = \frac{50000 - (1960.2 + 1998.2)}{50,000} \times 100$$

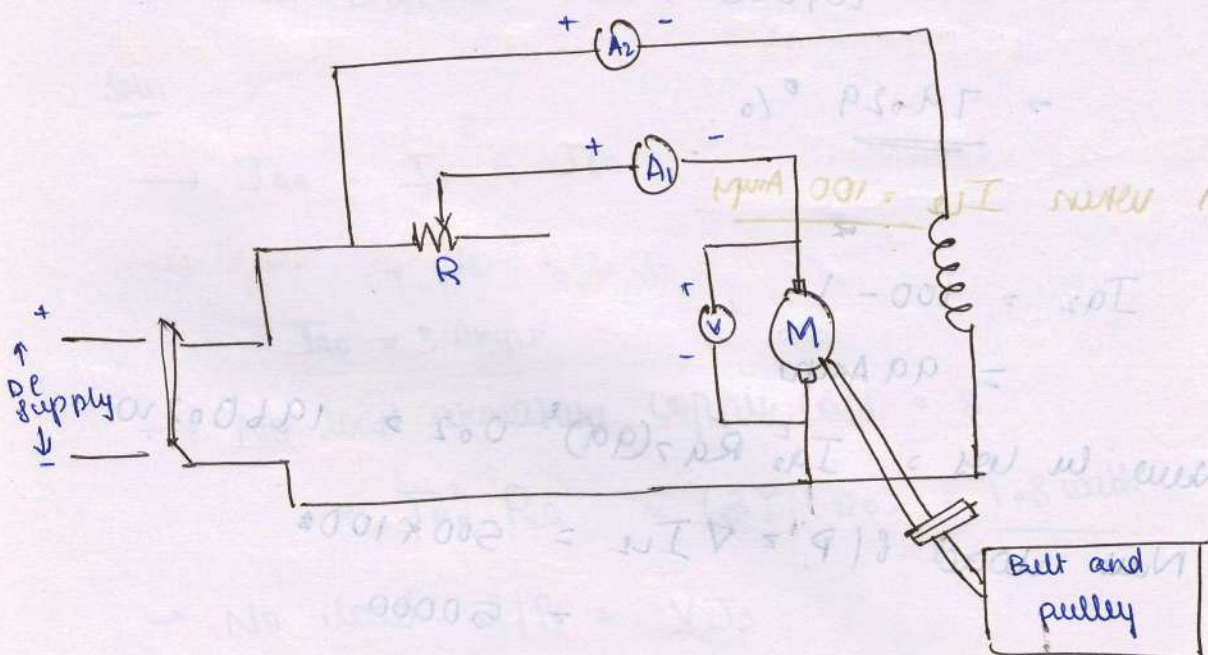
$$\% \eta = \underline{\underline{92.08\%}}$$

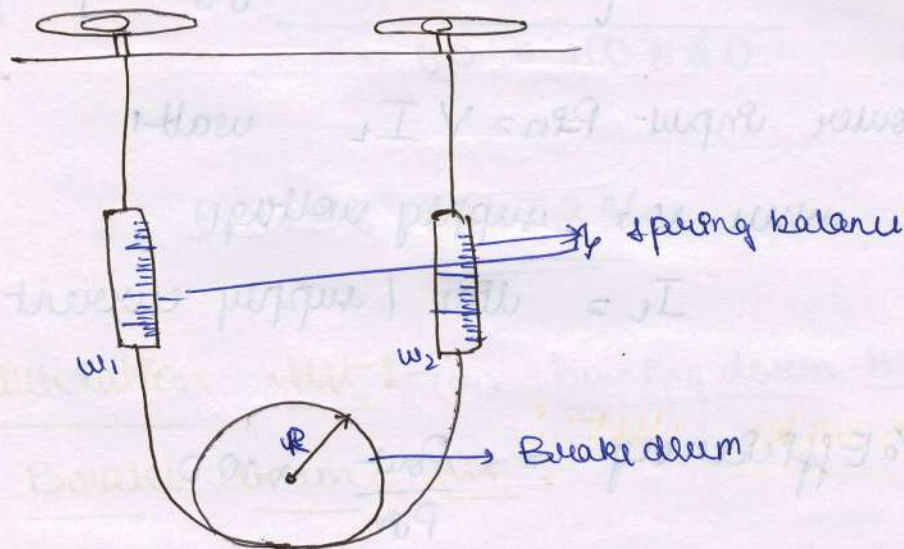
⇒ Brake test :-

This is direct method of testing a DC motor by means of a belt and pulley arrangement (brake drum).

By adjusting the tension of the belt, the load is adjusted to give various values of current.

Circuit diagram :-





Belt and pulley arrangement

Let R = radius of brake drum in metres

N = speed in rpm

w_1 = spring balance reading on tight side in Kg

w_2 = spring balance reading on slack side

∴ Net pull on the belt due to friction

is given by $= (w_1 - w_2)$ Kgs

Net pull $= 9.81 (w_1 - w_2)$ ~~newton~~ Newton

As the speed N and radius R is known, the shaft torque developed is given by

$$T_{sh} = \text{Net pull} \times R$$

$$= 9.81 (w_1 - w_2) \times R \text{ N-m}$$

The power o/p is given by

$$P_{out} = T_{sh} \times \omega \text{ watts}$$

$$P_{out} = \left[\frac{9.81 (w_1 - w_2) \times 2\pi NR}{60} \right] \text{ watts}$$

$$\text{Power Input } P_{in} = V I_L \text{ watts}$$

where $V =$ supply voltage

$I_L =$ line / supply current

$$\% \text{ Efficiency} = \frac{P_{out}}{P_{in}} \times 100$$

$$\% \eta = \frac{9.81 (w_1 - w_2) \times 2\pi NR \times 100}{60 \times V I_L}$$

Q1) In a brake test conducted on a DC shunt motor, the full load readings are observed as - tension of light wire $= 9.01 \text{ Kg}$, tension of lag wire $= 0.8 \text{ Kg}$, total current $= 10 \text{ A}$, supply voltage $= 110 \text{ V}$, speed $= 1320 \text{ rpm}$, $R = 7.05 \text{ cm}$. Calculate its full load efficiency.

Soln

Given

$$w_1 = 9.01 \text{ Kg}$$

$$w_2 = 0.8 \text{ Kg}$$

$$N = 1320 \text{ rpm}$$

$$I_L = 10 \text{ A}$$

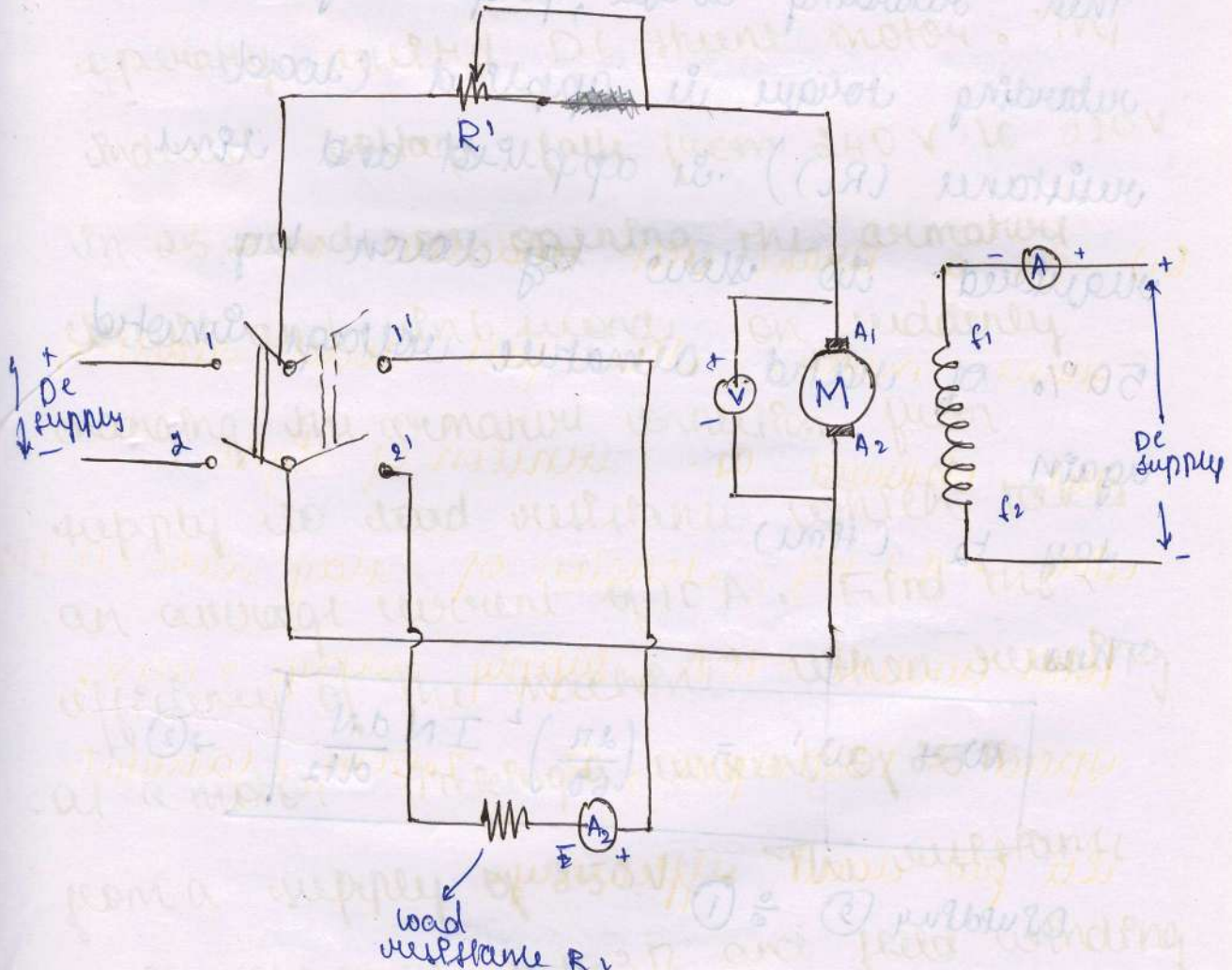
$$R = 7.05 \text{ cm} = 0.075 \text{ m}$$

$$\% \eta = \frac{9.81 (9.81 - 0.8) \times 2\pi \times 1320 \times 100}{60 \times 110 \times 40} \times 0.075$$

$$= \underline{\underline{76.73\%}}$$

⇒ Retardation test :- (or) Running down test :-

(or) Brake drum test :-



From this method we can get stray losses.

It is generally employed for DC shunt machines

If I is the moment of inertia of the

then

$$W = \left(\frac{2\pi}{60} \right)^2 I N \frac{dN}{dt_1} \quad \rightarrow (1)$$

In this method the time is noted for the machine to slow down by 50% of the rated armature voltage alone.

say t_1 (Hrs)

Then retarding torque, preferably electrical retarding torque is applied (load resistance R_L) is applied and time

required to slow down by 50% of rated armature voltage limited

again

say t_2 (Hrs)

Thus

$$W + W' = \left(\frac{2\pi}{60} \right)^2 I N \frac{dN}{dt_2} \quad \rightarrow (2)$$

Dividing (2) by (1)

$$\frac{W + W'}{W} = \frac{dt_1}{dt_2} \times \frac{dt_2}{dt_1} = \frac{dt_1}{dt_2}$$

$$W dt_2 + W' dt_2 = W dt_1$$

$$W' dt_2 = W dt_1 - W dt_2$$

$$w [dt_1 - dt_2] = w' dt_2$$

$$w = \frac{w' dt_2}{(dt_1 - dt_2)}$$

$$\Rightarrow w = \frac{w' t_2}{t_1 - t_2}$$

Q1) A retardation test is conducted on a separately excited DC shunt motor. The induced voltage falls from 240 V to 220 V in 25 seconds on opening the armature circuit and in 6 seconds on suddenly changing the armature connection from supply to load resistance which takes an average current of 10 A. Find the efficiency of the machine when running as a motor taking a current of 25 A from a supply of 250 V. The resistance of the armature is 0.3 Ω and field winding is 200 Ω .

Soln

Given: - $R_a = 0.6 \Omega$, $R_{sh} = 200 \Omega$

$$I_{avg} = 10 A$$

$$t_1 = 25 \mu s = 25 \times 10^{-6}$$

$$t_2 = 6 \mu s = 6 \times 10^{-6}$$

$$V_1 = 240 V$$

$$V_2 = 220 V$$

$$V_{avg} = \frac{V_1 + V_2}{2} = \frac{240 + 220}{2} = 230 V$$

Power absorbed by the load $= W'$

$$W' = V_{avg} \times I_{avg}$$

$$= 230 \times 10$$

$$= 2300 \text{ watts}$$

$$W = \frac{W' \times t_2}{t_1 - t_2}$$

$$= \frac{2300 \times 6}{25 - 6}$$

$$= 726.31 \text{ watts} = \text{steady loss (constant)}$$

Power i/p to the motor $= V I_d$

$$P = 250 \times 25$$

$$= 6250 \text{ watts}$$

$$\rightarrow \text{Copper loss} = I_a^2 R_a$$

$$= (25)^2 \times 0.3$$

$$I_a = I_L - I_{sh}$$

$$= I_L - \frac{V}{R_{sh}}$$

$$= 25 - \frac{250}{200}$$

$$I_a = 23.75$$

$$\text{Armature copper loss} = I_a^2 R_a$$

$$= (23.75)^2 \times 0.3$$

$$= 169.21 \text{ watt}$$

$$\text{Field copper loss} = V_{sh} I_{sh}$$

$$(or) I_{sh}^2 R_{sh}$$

$$= \left(\frac{250}{200} \right)^2 \times 200$$

$$= 312.5 \text{ watt}$$

$$\text{Total loss} = \text{Armature loss} + \text{Field loss} + \text{Stray loss}$$

$$= 169.21 + 312.5 + 726.03$$

$$= 1208.02 \text{ watt}$$

$$\% \eta_m = \frac{\text{IP} - \text{loss}}{\text{IP}} \times 100 = \frac{6250 - 1208.02}{6250} \times 100$$

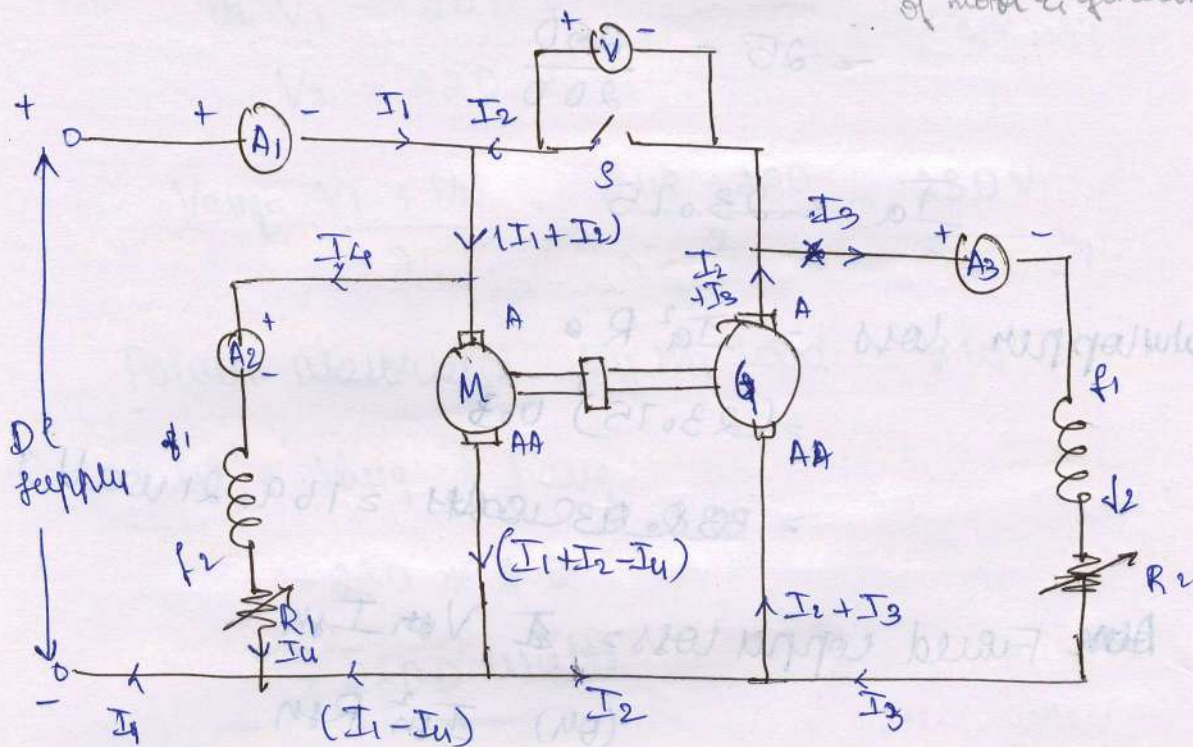
$$= 80.67 \%$$

⇒ Hopkinson's test :-

(or) Regenerative test :-

(or) back to back test :-

without shunt
 (M) (G) current
 of generat does not
 have path for
 flowing
 (V) made up
 to avoid circulating
 currents and
 allow synchronization
 of motor & generator



Motor i/p electric through generator
 Generator i/p mechanic through motor

∴ Back to back test-

over exciting - Generator } through rheostat
 under exciting - motor }

- Let V = supply voltage

I_1 = current drawn from the supply

I_2 = current supplied by the generator

I_3 = Exciting / field current of the generator

field current = Exciting current

I_f = Exciting or field current of motor

R_a = Resistance of each armature of each machine

→ Armature copper loss in the generator

$$= \underline{(I_2 + I_3)^2 R_a \text{ watts}}$$

→ Field copper loss in the generator

$$= \underline{V I_3 \text{ watts}}$$

→ Armature copper loss in the motor = $\underline{(I_1 + I_2 - I_f)^2 R_a \text{ w}}$

Field copper loss in the motor = $\underline{V I_f \text{ watts}}$

The total losses in the generator and motor are equal to power supplied by the mains

∴ Power drawn from the supply = $\underline{V I_1 \text{ watts}}$

∴ Total stray losses = Power I/P - Total cu losses

$$W_s = V I_1 - \left[(I_2 + I_3)^2 R_a + V I_3 + (I_1 + I_2 - I_f)^2 R_a + V I_f \right]$$

∴ Stray loss per machine = $\underline{W_s / 2}$

(ii) Ana

(P) Motor efficiency :-

$$\% \eta_m = \frac{\text{P/P} - \text{losses}}{\text{P/P}} \times 100$$

→ Power s/p to the motor = $V(I_1 + I_2)$ watts

Total loss of the motor =

$$= (I_1 + I_2 - I_u)^2 R_a + V I_u + Ws/2$$

$$\% \eta_m = \frac{V(I_1 + I_2) - [(I_1 + I_2 - I_u)^2 R_a + V I_u + Ws/2]}{V(I_1 + I_2)} \times 100$$

(iii) Generator efficiency :-

consider electric parameters

motor = P/P - electric
generator = O/P - electric

$$\% \eta_m = \text{P/P}$$

$$\% \eta_g = \text{O/P}$$

$$\% \eta_g = \frac{\text{O/P}}{\text{O/P} + \text{losses}} \times 100$$

Power o/p of the generator = $V I_2$ watts

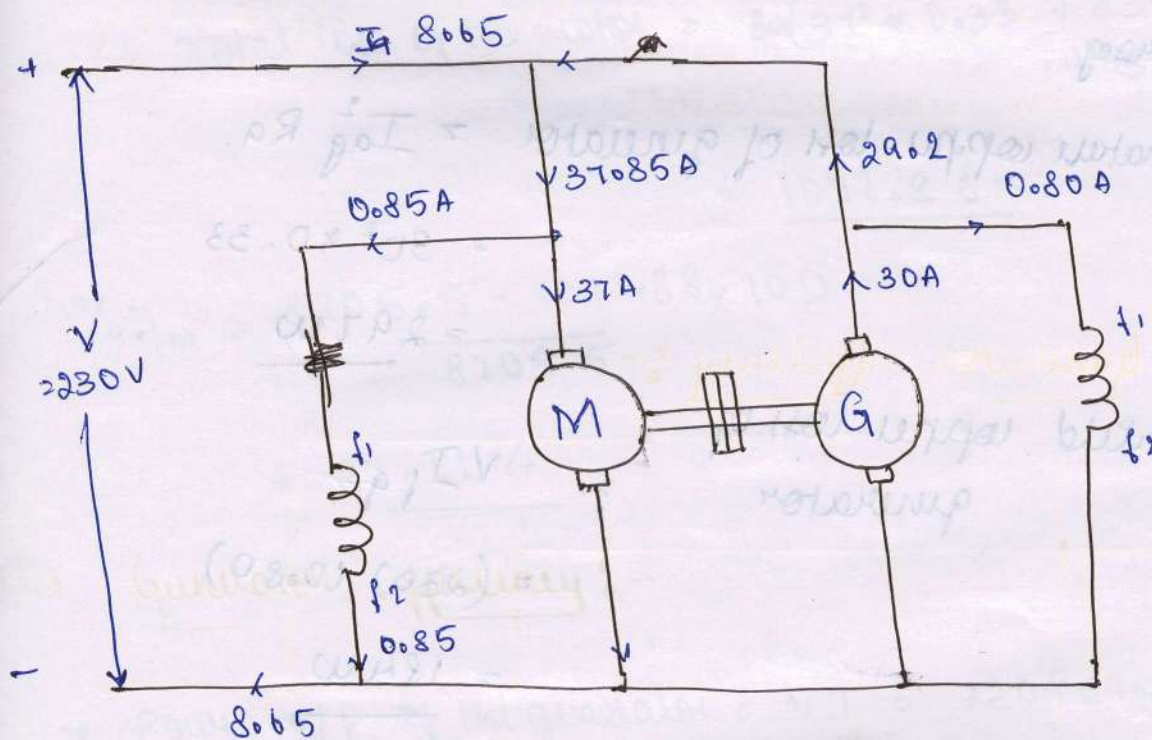
Total loss of generator

$$= (I_2 + I_3)^2 R_a + V I_3 + Ws/2 \text{ watt}$$

$$\% \eta_g = \frac{V I_2}{[(I_2 + I_3)^2 R_a + V I_3 + Ws/2] + [V I_2]} \times 100$$

~~Ex 1)~~ Following results were obtained during
 Hopkinson's on two similar DC shunt
 machines of 230V, Armature currents 37A
 and 30A, field current 0.85A and 0.80A
 respectively calculate the efficiency of the
 machines if each has ~~an~~ armature resistance
 of 0.33 ohms

Soln



Given

Armature current = 37A, 30A

field current = 0.85A, 0.80A

$R_a = 0.33 \Omega$

Soln

// (method - 1 - some using)

For motor - generator set

→ Armature copper loss of generator = $I_a^2 R_a$

$= 37^2 \times 0.33$

$= \underline{451.77 \text{ W}}$

→ Field copper loss of motor = $V I_{fm}$

$= 220 \times 0.85$

$= \underline{195.5 \text{ W}}$

Sketch

→ Armature copper loss of generator = $I_a^2 R_a$

$= 30^2 \times 0.33$

$= \underline{297 \text{ W}}$

→ Field copper loss of generator = $V I_{fg}$

$= (230) (0.80)$

$= \underline{184 \text{ W}}$

Loop

→ Total power S/P = VI

$= 230 \times 8.65$

$= \underline{1989.5 \text{ W}}$

$$\rightarrow \text{Stray loss} = W_s = I/P - \text{total w losses}$$

$$= 1989.5 - (451.77 + 195.5 + 297 + 1847)$$

$$= \underline{861.23 \text{ W}}$$

$$\rightarrow \text{Stray loss per machine} = 430.615 \text{ W}$$

(i) motor efficiency % -

$$\rightarrow \text{Power I/P to the motor} = 230 \times 37.5$$

$$= 8705.5 \text{ W}$$

$$\rightarrow \text{Total loss of the motor} = 3(37^2 \times 0.33 + 230 \times 0.85) + 430.615$$

$$= 1077.88 \text{ W}$$

$$\% \eta_m = \frac{8705.5 - 1077.88}{8705.5} \times 100$$

$$= \underline{87.61\%}$$

(ii) Generator efficiency % -

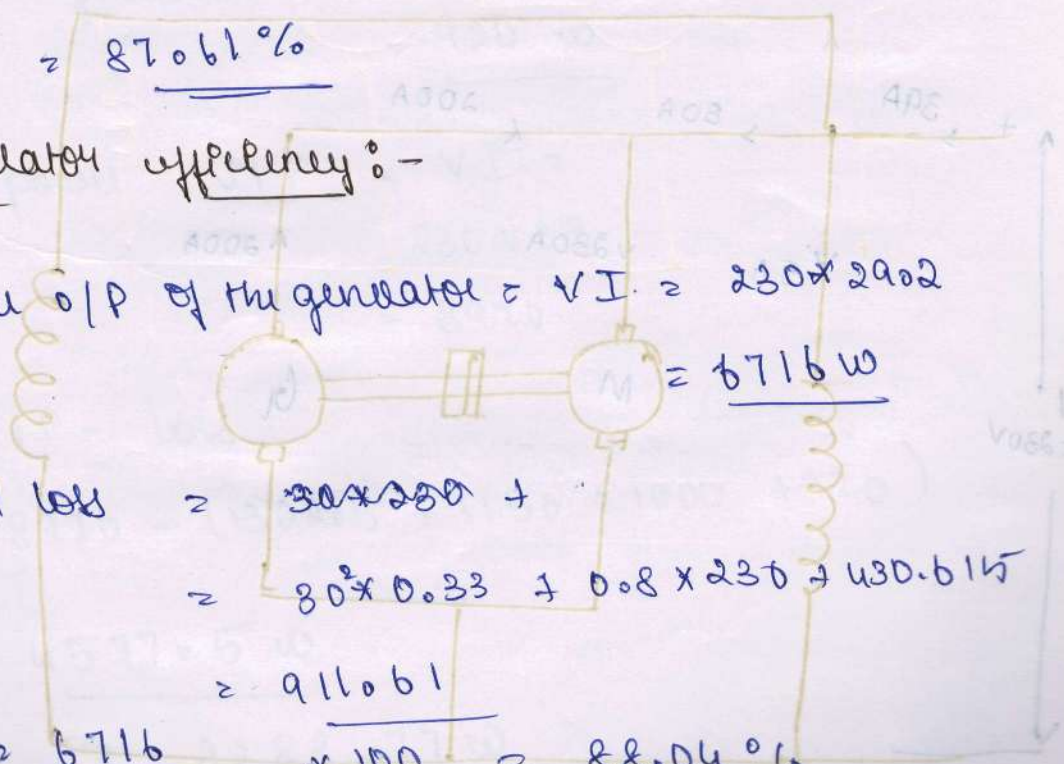
$$\rightarrow \text{Power O/P of the generator} = V I = 230 \times 29.2$$

$$= \underline{6716 \text{ W}}$$

$$\rightarrow \text{Total loss} = 30 \times 230 + 30^2 \times 0.33 + 0.8 \times 230 + 430.615$$

$$= 911.61$$

$$\% \eta_g = \frac{6716}{6716 + 911.61} \times 100 = \underline{88.04\%}$$



Q2) Two identical DC shunt machines when test by Hopkinson's method gave the following results



(i) Line voltage 230 V

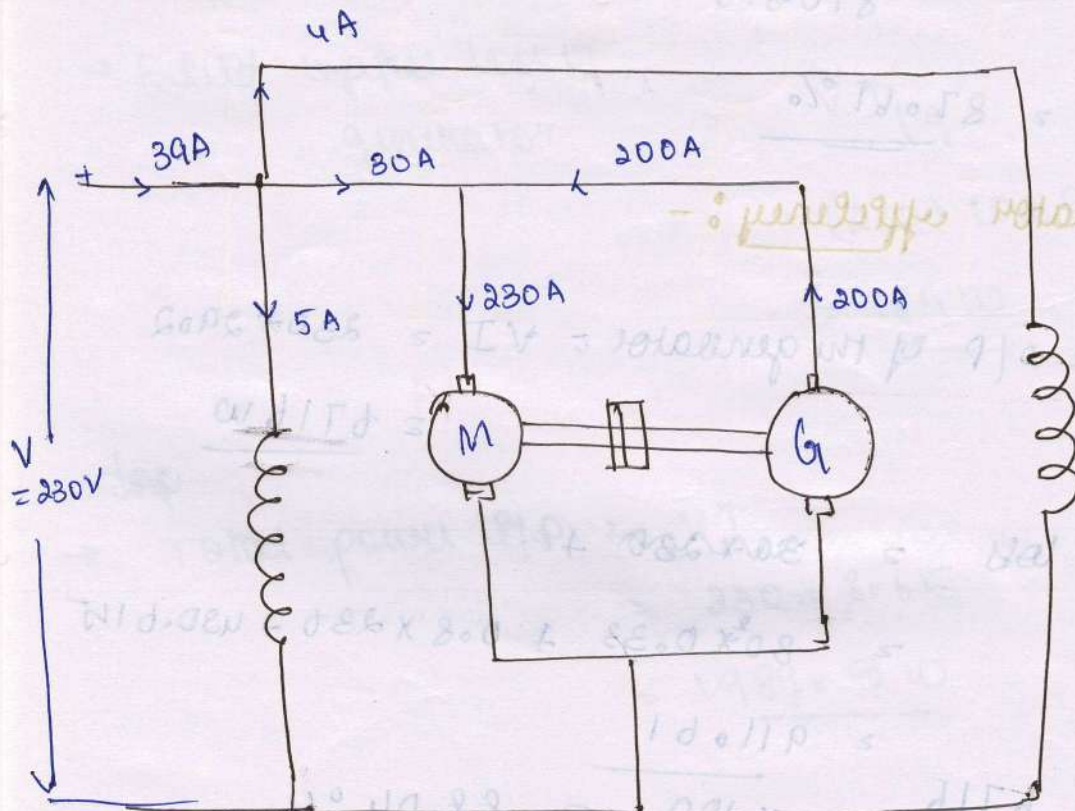
(ii) Line current including field current = 30 A

(iii) Motor armature current as 230 A

Soln (iv) Field current 5 A & 2 A separately

(v) The armature resistance of each machine 0.025Ω . Calculate the efficiency of both the machines

// (Method - 2) → Same thing



Soln

For motor generator set

$$\begin{aligned}\rightarrow \text{Armature Cu loss of motor} &= I_a^2 R_a \\ &= (230)^2 \cdot 0.025 \\ &= \underline{13220.5 \text{ W}}\end{aligned}$$

$$\begin{aligned}\rightarrow \text{Field Cu loss motor} &= V I_{fm} \\ &= 230 \times 5 \\ &= \underline{1150 \text{ W}}\end{aligned}$$

$$\begin{aligned}\rightarrow \text{Armature Cu loss of generator} &= I_a^2 R_a \\ &= (200)^2 \cdot 0.025 \\ &= \underline{1000 \text{ W}}\end{aligned}$$

$$\begin{aligned}\rightarrow \text{Field Cu loss of generator} &= 230 \times 4 \\ &= \underline{920 \text{ W}}\end{aligned}$$

$$\begin{aligned}\rightarrow \text{Total power } P/P &= V I = \\ &= 230 \times 39 \\ &= 8970\end{aligned}$$

$$W_s = P/P - \text{loss}$$

$$= 8970 - (13220.5 + 1150 + 1000 + 920)$$

$$= \underline{4577.5 \text{ W}}$$

$$W_s / 2 = \underline{2288.75 \text{ W}}$$

(8) Motor efficiency

$$\rightarrow \text{Power i/p} = 230 \times (230 + 5)$$

$$= 54050 \text{ W}$$

$$\rightarrow \text{Total loss} = (230^2 \times 0.0025) + (230 \times 5) + 2288.75$$

$$= 4761.25 \text{ W}$$

$$\% \eta_m = \frac{54050 - 4761.25}{54050} \times 100$$

$$= 91.19\%$$

(ii) Generator efficiency

$$\rightarrow \text{Power o/p} = 230 \times 200$$

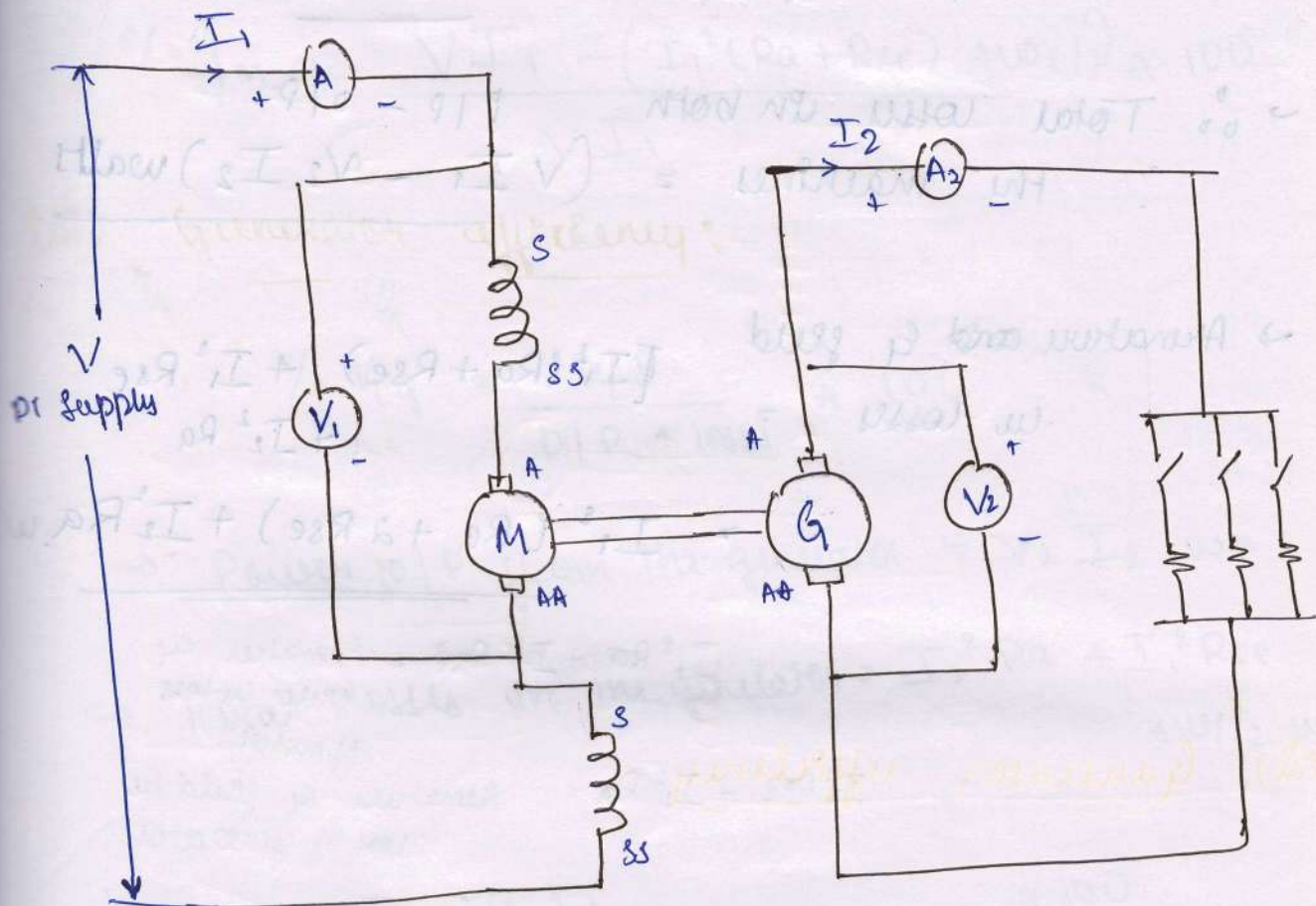
$$= 46000 \text{ W}$$

$$\rightarrow \text{Total loss} = (200^2 \times 0.0025) + (230 \times 4) + 2288.75$$

$$= 4208.75 \text{ W}$$

$$\% \eta_g = \frac{46000}{46000 + 4208.75} = 91.61\%$$

⇒ Field test on DC series machines:-



Let V = supply voltage

I_1 = Current drawn by the motor

I_2 = Current supplied by the generator

V_1 = Voltage drop across motor

V_2 = Voltage drop across generator

R_a & R_{se} = Armature and series field resistance of each machine.

→ Power drawn from the supply = $V I_1$ watts

→ output of the generator = $V_2 I_2$ watts

→ ∴ Total loss in both machines = $(V I_1 - V_2 I_2)$ watts

→ Armature and field

cu losses =

$$[I_1^2 (R_a + R_{se}) + I_1^2 R_{se} + I_2^2 R_a]$$

$$= I_1^2 (R_a + 2 R_{se}) + I_2^2 R_a$$

$I_1^2 R_a + I_1^2 R_{se}$ = Armature & field cu loss of motor

$I_1^2 R_{se} + I_2^2 R_a$ = Armature & field cu loss of generator

→ stray loss = Total loss - cu loss

$$W_s = (V I_1 - V_2 I_2) - [I_1^2 (R_a + 2 R_{se}) + I_2^2 R_a]$$

Stray loss per machine = $W_s / 2$

(P) Motor efficiency % -

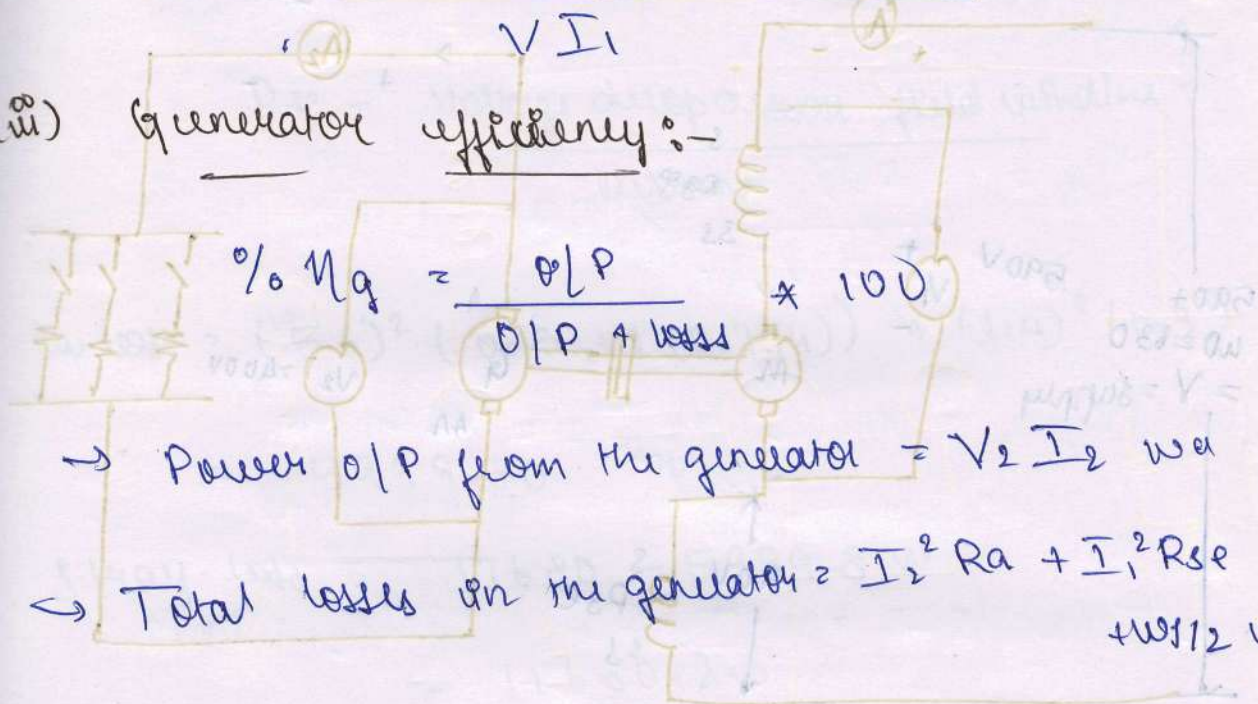
$$\% \eta_m = \frac{\text{P/P} - \text{losses}}{\text{P/P}} \times 100$$

→ Power P/P to the motor = $V I_1$ watts

→ Total losses in the motor = $I_1^2 R_a + I_1^2 R_{se} + W_s/2$
 $= I_1^2 (R_a + R_{se}) + W_s/2$ watt

$\% \eta_m = \frac{V I_1 - (I_1^2 (R_a + R_{se}) + W_s/2)}{V I_1} \times 100$

(iii) Generator efficiency:-



$\% \eta_g = \frac{O/P}{O/P + \text{losses}} \times 100$

→ Power O/P from the generator = $V_2 I_2$ watt

→ Total losses in the generator = $I_2^2 R_a + I_1^2 R_{se} + W_s/2$ watt

$\% \eta_g = \frac{V_2 I_2}{V_2 I_2 + I_2^2 R_a + I_1^2 R_{se} + W_s/2} \times 100$

Q1) A test on two coupled similar fram way motors, with their field ~~is~~ connected in series, gave the following result when one machine acted as a motor and the other as a generator.

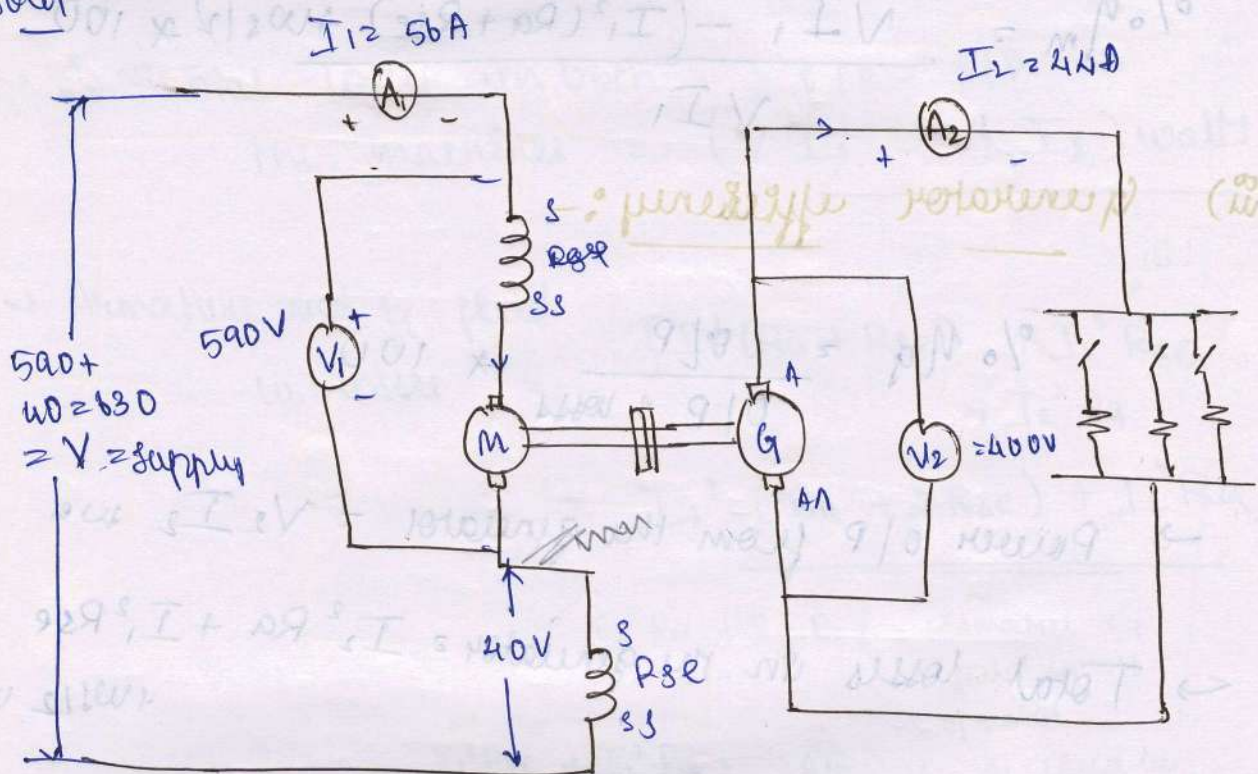
(a) Motor:- $I_a = 56$ A, Armature Voltage = 590 V,
 Voltage drop across field winding = 40 V

(b) Generator:- $I_a = 44$ A, Armature voltage = 400 V,
 voltage drop across field = 40 V

Resistance of each armature = 0.03Ω

Calculate the efficiency of the motor and the generator.

Soln



Armature voltage = V_m + V_g (both include) $\rightarrow$ not single armature also

$\rightarrow$ Total supply voltage = $V_g = 590 + 40 = 630$

$\rightarrow$ Supply current = $I_1 = 56A$

$\rightarrow$ Total power $i/p = VI = 630 \times 56 = 35280 W$

$\rightarrow$ o/p of generator, $V_2 \times I_2 = 400 \times 44 = 17600 W$

$\rightarrow$ Total loss = $i/p - o/p = 35280 - 17600$

$= 17680 W$

$$\rightarrow \text{cu loss} = [I_1^2 (R_a + 2R_{se}) + I_2^2 R_a]$$

$$= (56)^2 (0.3 + 2(0.3))$$

$$R_{se} = \frac{V_{af}}{I_f} = \frac{590}{56} = \frac{40}{56} = 0.714$$

$$R_{se} = \frac{\text{Voltage drop across field winding}}{\text{current}}$$

$$\text{cu loss} = (56)^2 (0.3 + 2(0.714)) + (44)^2 (0.3)$$

$$= 5999.80$$

$$\text{ stray loss} = 17680 - 5999.8 \text{ w}$$

$$= 11680.2 \text{ w}$$

$$\text{ws} / 2 = 5840.1 \text{ w}$$

(P) motor efficiency :-

$$\% \eta_m = \frac{P/P - \text{loss}}{P/P} \times 100$$

$$\rightarrow \text{Power P/P to motor} = V I_1 = 590 \times 56$$

$$= 33040 \text{ w}$$

$$\rightarrow \text{losses} = I_1^2 R_a + I_1^2 R_{se} + \text{ws} / 2$$

$$= 56^2 (0.3) + 56^2 (0.714) + 5840.1$$

$$= 9020.00 \text{ w}$$

$$\% \eta = \frac{33040 - 9020}{33040} \times 100$$

$$= 72.7\%$$

→ (iii) Generator efficiency :-

$$\eta_g = \frac{O/P}{O/P + P/P} \times 100$$

$$\rightarrow \text{Power O/P} = V_2 I_2 = 400 \times 44$$

$$= 17600 \text{ W}$$

$$\rightarrow \text{Total loss} = I_2^2 R_a + I_1^2 R_{se} + W_{s/l2}$$

$$= (44^2)(0.3) + (156)^2(0.0114) + 5839.7$$

$$= 8659.16$$

$$\% \eta = \frac{17600}{17600 + 8659.16} \times 100$$

$$= 67.02\%$$

DE supply = 220V

1 ϕ AC $\rightarrow$ 230V, 50Hz

3 ϕ AC $\rightarrow$ 415/430/440, 50Hz

Transf

Generator
motor

Transf

Generator
motor

$$\rightarrow V_{ph} = \frac{V_L}{\sqrt{3}} ; I_L = I_{ph}$$

Synchronous speed
 $\rightarrow$ synchronous speed

$$N_s = \frac{120f}{P}$$

Induction motor
 $\rightarrow$ sub-synchronous speed
 $\neq N_s$

RMF $\rightarrow$ rotates at N_s
in 3 ϕ m/c has
rotates at N_s

RMF $\rightarrow$ generated by
stator winding

D.C machine $\rightarrow$ physical pole
 A.C machine $\rightarrow$ imaginary pole
 Im $\rightarrow$ imaginary pole

Three - phase induction motor:-

Rmy means
 the effect
 of imaginary
 pole

rotates at
 synchronous
 speed.

Stator $\rightarrow$ 3 ϕ (compulsory) $\rightarrow$ Y or Δ conn
 same type of winding Y or Δ

Rotor -

$\Phi_R = \Phi_m \sin \theta$ $\rightarrow$ always

$\Phi_Y = \Phi_m \sin(\theta - 120^\circ)$

$\Phi_B = \Phi_m \sin(\theta - 240^\circ)$

By
 instantaneous
 sine wave
 eqns

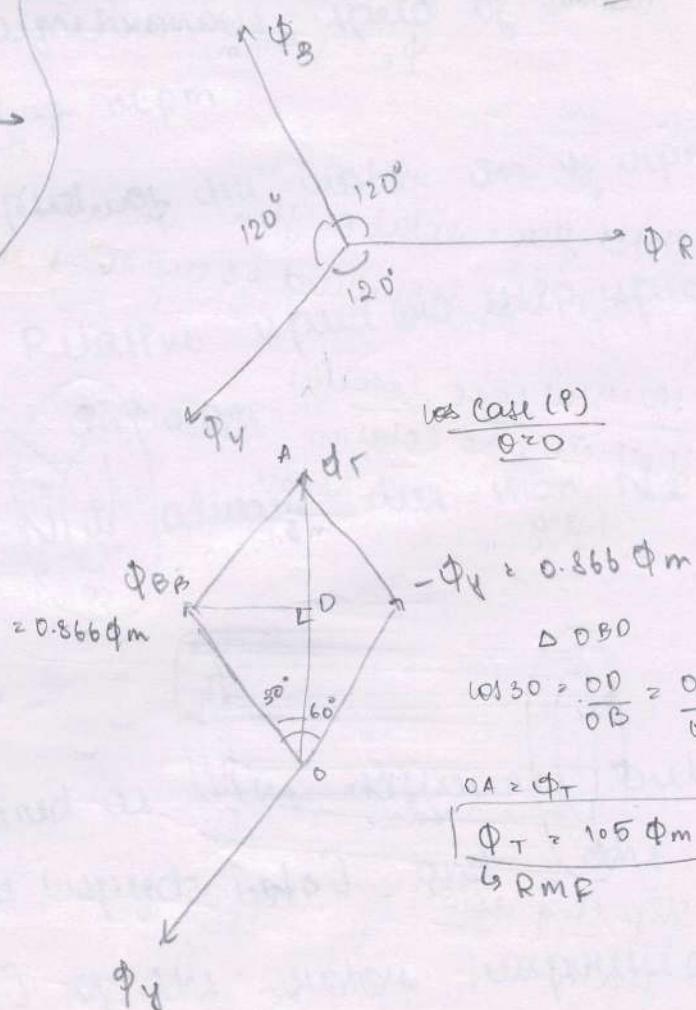
// works on electromagnetic induction
 RMP
 stator -
 Zero crossing at
 $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ$

$\Phi_R = \Phi_m \sin 0 = 0$

$\Phi_Y = \Phi_m \sin(-120^\circ) = -0.866 \Phi_m$

$\Phi_B = \Phi_m \sin(-240^\circ) = +0.866 \Phi_m$

Assumed the
 direction phase diagram



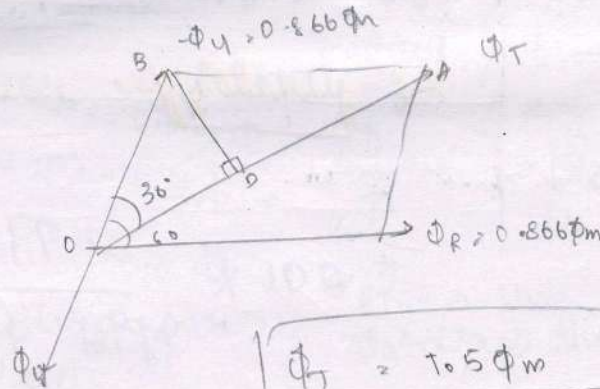
11/11/19

case 2 $\theta = 60^\circ$

$$\Phi_R = \Phi_m \sin 60 = 0.866 \Phi_m$$

$$\Phi_Y = \Phi_m \sin (60 - 60) = 0$$

$$\Phi_B = \Phi_m \sin (60 + 60) = 0$$



$$\cos 30 = \frac{OB}{\Phi_T} = \frac{0.866 \Phi_m}{\Phi_T}$$

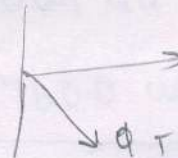
$$\Phi_T = 1.5 \Phi_m$$

$$\boxed{\Phi_T = 1.5 \Phi_m}$$

11/11/19

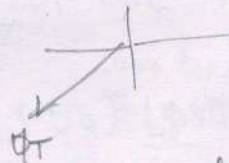
case (3) $\theta = 120^\circ$

$$\Phi_T = 1.5 \Phi_m$$



case (4) $\theta = 180^\circ$

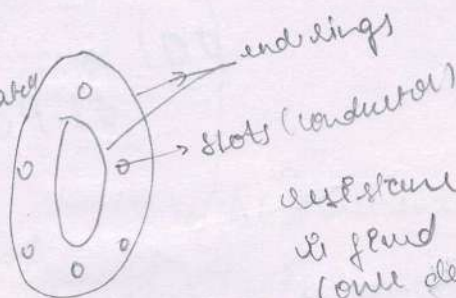
$$\Phi_T = 1.5 \Phi_m$$



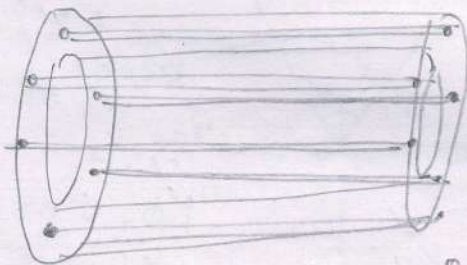
Stator flux + rotor flux interact to produce force and torque is produced $\Rightarrow$ and rotor conductors starts rotating

(i)

salient pole rotor (or)
short circuited rotor (or) unexcited
(or) Al base

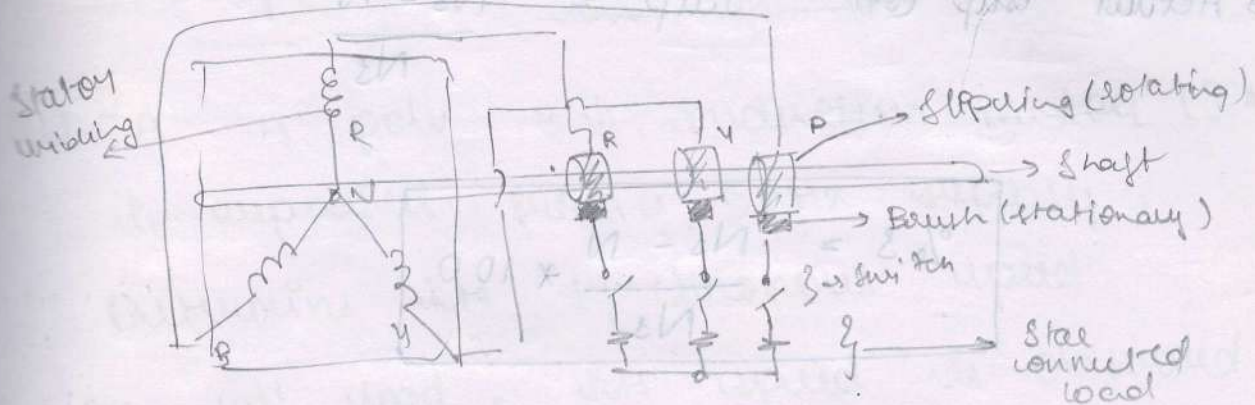


resistance value is fixed (only designed it is fixed) not external resistance can be added



(ii)

slip ring rotor: - slip ring and brush assembly is stationary



Stator flux (RMP) + Rotor flux $\rightarrow$ emf
 emf gets induced in rotor
 frequency is off

* External resistance can be added to central rotor & frequent maintenance is necessary

$\Rightarrow$ Slip of an induction motor :- (S) :-
 (or) (Absolute slip)

$\rightarrow$ Let N_s = synchronous speed of the RMP in rpm

N = speed of the motor in rpm

$N_s - N$ = Relative speed or slip speed in rpm

$N < N_s$ = N is always less than N_s

Definition :-

It is defined as the difference between the synchronous speed [N_s] and the actual speed [N] of the motor expressed as a fraction of synchronous speed N_s

→ Actual slip (or) $s_{\text{slip}} = \frac{N_s - N}{N_s}$

$$\% s = \frac{N_s - N}{N_s} \times 100$$

⇒ Actual speed (N) in terms of N_s:-

$$(s \times N_s) = (N_s - N)$$

$$sN_s - N_s = -N$$

$$N_s (s - 1) = -N$$

$$N = N_s (1 - s)$$

* At start

$$N = 0$$

$$0 = N_s (1 - s)$$

$$1 - s = 1$$

$$s = 1$$

[maximum value that the induction motor can attain i.e., $s = 1$]

s is maximum at start]

- The value of slip is maximum i.e., $s = 1$ only at start, and goes on reducing as the motor starts

- slip can never be zero

Q1) A 4 pole 3 ϕ induction motor (IM) is supplied from a 50Hz supply. Determine its synchronous speed on full load, its speed is observed to be 1410 rpm calculate its full load slip.

Soln

$$P = 4$$

$$f = 50 \text{ Hz}$$

$$N = 1410 \text{ rpm}$$

To find = $s = ?$

$$N_s = \frac{120f}{P}$$
$$= \frac{120 \times 50}{4}$$

$$= \underline{1500}$$

$$s = \frac{N_s - N}{N_s}$$

$$= \frac{1500 - 1410}{1500}$$

$$s = \underline{0.06}$$

(or) $s\% = 6\%$

it can't be more than 1 and cannot be zero and (values from 10-15%)

→ Effect of slip on the motor parameters:-

- slip affects ~~some~~ the frequency of the motor induced emf and hence some motor parameters also get affected.

→ the following motor parameters get affected due to slip:-

- (i) Rotor frequency
- (ii) Magnitude of motor induced emf
- (iii) Rotor reactance
- (iv) Rotor power factor
- (v) Rotor current

* (i) Effect on motor frequency:- (f_r):-
WKT $f_r = \text{motor frequency}$

$$\text{speed of rpm} = \frac{120f}{P} = N_s \rightarrow \text{① (at start)}$$

$f = \text{frequency of supply}$

At start $N = 0$, $s = 1$

- Thus stationary motor has maximum relative motion w.o.t RMF
- Hence maximum emf gets induced at start and frequency of this induced emf is same as supply
- As the motor ^(rotor) starts running with a speed of 'N' rpm, the relative motion between the motor w.o.t 'RMF' decreases and becomes equal to the slip speed $[N_s - N]$
- Hence in running condition, the magnitude of induced emf decreases and so does the frequency (decreases)
- If f_r = frequency of motor induced emf in running condition at slip speed $N_s - N$ then

$$N_s - N = \frac{120 f_r}{P} \rightarrow (2)$$

→ running condition

Dividing eqn (2) by eqn (1)

$$\frac{N_s - N}{N_s} = \frac{120 f_r / P}{120 f / P}$$

$$s = \frac{f_s}{f}$$

$$\therefore \boxed{f_s = sf}$$

At start $s = 1$, $N = 0$ [slip is max at start]

$$\therefore \boxed{f_s = f}$$

→ (iii) Effct on magnitude of motor induced emf :-
(Exam)

• W.K.T at start $N = 0$, $s = 1$ and relative speed is maximum
(or) motion

$\therefore$ Induced emf is also maximum

Let this emf be E_2

i.e., $E_2 = \frac{\text{motor induced emf per phase at stand still}}{}$

• As the motor starts running relative speed decreases, the induced emf in the motor also decreases proportional to the slip speed $[N_s - N]$

$$E_2 \propto N_s \rightarrow \textcircled{1}$$

$$E_{2m} \propto [N_s - N] \rightarrow \textcircled{2}$$

E_{2m} = induced emf in motor at running condition

$\therefore$ dividing $\textcircled{2}$ by $\textcircled{1}$

$$\frac{E_{2m}}{E_2} = \frac{N_s - N}{N_s}$$

$$\frac{E_{2m}}{E_2} = s$$

$$E_{2m} = s E_2$$

At start $s = 1$, $N = 0$

$$E_{2m} = E_2$$

→ (iii) Effect on motor impedance: — (X_{2m})

(It will not have effect on resistance // only on reactance)

$(R + jX_L)$ varies
 $\downarrow$
 constant

$\therefore$ Resistance is independent of frequency

$X_L = 2\pi f L$
 $\downarrow$
 change

$\therefore X_L$ changes as frequency changes

Let R_2 = motor resistance per phase on standstill

X_2 = motor reactance per phase on standstill

→ At standstill, $N=0$, $s=1$

w.k.T → $f_u = f$

at standstill

$$f_u = f \quad [s=1]$$

f L_2 is the motor inductance per phase

then

$$\left[\begin{aligned} X_2 &= 2\pi f L_2 \\ &= 2\pi f L_2 \end{aligned} \right] \begin{matrix} \Omega / \text{phase} \\ \Omega / \text{phase} \end{matrix} \quad \text{--- (1)}$$

In running condition

$$X_{2u} = 2\pi f_u L_2$$

w.k.T

$$f_u = s f \quad \text{in running condition}$$

$$\therefore \left[X_{2u} = 2\pi s f L_2 \right] \Omega / \text{phase} \quad \text{--- (2)}$$

where X_{2u} is motor reactance per phase

in running condition

$$\therefore X_{2u} = s X_2$$

$$X_{2u} = \underbrace{(2\pi f L_2)}_{X_2} s$$

from eqn (1)

$$\boxed{X_{2u} = s X_2}$$

The motor resistance per phase remains unchanged because it is independent of frequency

→ Thus the motor impedance per phase is Z_2 at standstill is

$$\boxed{Z_2 = R_2 + jX_2} \quad \Omega / \text{phase}$$

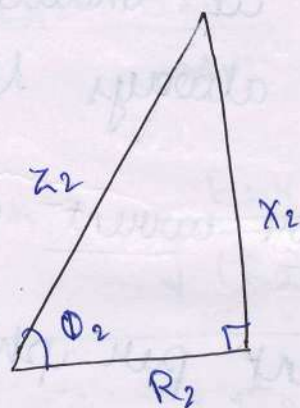
→ Thus the motor impedance per phase under running condition is

$$\boxed{Z_{2u} = R_2 + jX_{2u}} \quad \Omega / \text{phase}$$

$$\therefore \boxed{Z_{2u} = R_2 + jSX_2}$$

→ (iv) Effect on motor power factor:-

At standstill

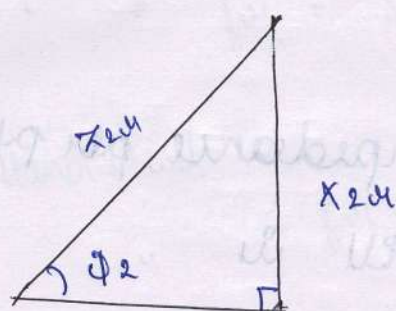


(Impedance triangle)

$$\cos \phi_2 = \frac{R_2}{Z_2}$$

$$\cos \phi_2 = \frac{R_2}{R_2 + jX_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

At running condition



$$\cos \phi_{2w} = \frac{R_2}{Z_{2w}}$$

$$= \frac{R_2}{R_2 + jX_{2w}}$$

$$= \frac{R_2}{R_2 + j(sX_2)}$$

$$= \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

As the motor winding is inductive, the motor power factor is always lagging

→ (ii) Effect on motor current :-

Let I_2 = motor current per phase on standstill

I_2 depends on the magnitude of E_2 and Z_2

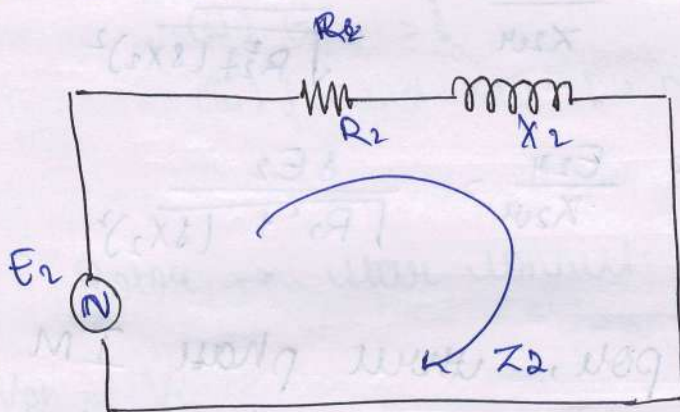
Z_2 is impedance at standstill

When E_2 = motor induced emf per phase on standstill

$$\therefore I_2 = \frac{E_2}{Z_2} \quad \text{per phase}$$

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + (X_2)^2}} \quad \text{per phase}$$

→ The equivalent motor circuit on standstill is as shown

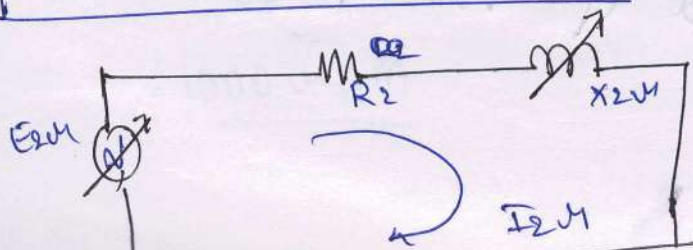


Let I_{2M} = motor current per phase under running condition.

$$I_{2M} = \frac{E_{2M}}{Z_{2M}} \quad \text{per phase}$$

$$I_{2M} = \frac{E_{2M}}{\sqrt{(R_2)^2 + (sX_2)^2}} \quad \text{per phase}$$

$$I_2 = \frac{s E_2}{\sqrt{(R_2)^2 + (sX_2)^2}} \quad \text{per phase}$$



⇒ Effect of slip on motor parameters:-

(i) $f_m = s f$

(ii) $E_{2m} = s E_2$

(iii) $X_{2m} = s X_2$

and

$$Z_{2m} = \sqrt{R_2^2 + (s X_2)^2}$$

(iv) $\cos \phi_{2m} = \frac{R_2}{Z_{2m}} = \frac{R_2}{\sqrt{R_2^2 + (s X_2)^2}}$

(v) $I_{2m} = \frac{E_{2m}}{Z_{2m}} = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}}$

Q1) In a 6 pole, three phase IM 50 Hz with star connected motor, the motor resistance per phase is 0.3 ohm the reactance at standstill is 1.5 Ω per phase an supply the slip rings on open circuit is 175 V. Calculate

(i) Slip at a speed of 960 rpm

(ii) Rotor emf per phase

(iii) Rotor frequency and reactance at a speed of 950 rpm.

Soln Given :-

$$P = 6$$

$$f = 50 \text{ Hz}$$

$$R_2 = 0.3 \Omega / \text{phase}$$

$$X_2 = 1.5 \Omega / \text{phase}$$

$$E_{2(\text{rms})} = 175 \text{ V}$$

To find :-

$$(i) s = ? \rightarrow N_s = 2600 \text{ rpm}$$

$$(ii) E_{2s} = ?$$

$$(iii) f_r \text{ and } X_{2s} = ? \rightarrow N_s = 2500 \text{ rpm}$$

Soln

Rotor $\rightarrow$ star connect

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$E_{ph} = \frac{E_L}{\sqrt{3}}$$

$$= \frac{175}{\sqrt{3}}$$

$$= 101.036 \text{ V}$$

$$\rightarrow N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{6}$$

$$= \underline{\underline{1000 \text{ rpm}}}$$

$$N = 960 \text{ rpm}$$

$$\rightarrow s = \frac{N_s - N}{N_s}$$

$$= \frac{1000 - 960}{1000}$$

$$s = \underline{\underline{0.04}}$$

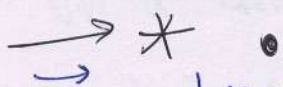
Always E
should in
phase

$$\rightarrow E_{2m} = 101.036 \text{ V}$$

$$E_{2m} = s \times E_2$$

$$= 0.04 \times 101.036$$

$$E_{2m} = \underline{\underline{4.041 \text{ V / phase}}}$$



$$f_m = s f$$

$$f_m = 0.04 \times 50$$

$$f_m = \underline{\underline{2.05}}$$

$$\rightarrow X_{2m} = s X_2$$

$$= 0.04 \times 0.15$$

$$= \underline{\underline{0.006 \Omega \text{ / phase}}}$$

→ New value of speed $N = 950 \text{ rpm}$

$$s = \frac{1000 - 950}{1000} = \underline{\underline{0.05}}$$

→ Induction motor as a rotating transformer:-

Transformer	Induction machine
1) Rot Alternating flux is produced	Rotating flux (RMP)
2) There is no air gap between the windings	There is air gap between winding (stator and rotor)
3) Frequency of induced emf and current in primary and secondary is same	The frequency of stator rotor emf and current depends on slip and slip depends load on motor except starting condition
4) The entire energy present in the secondary circuit is in electrical form	Some ^{minor} part of the energy in the motor circuit is electrical form and major part of the energy is in mechanical form

⇒ Transformation ratio in 3 ϕ IM:-

→ At start $N=0$, $s=1$

$$\text{where } \frac{E_2}{E_1} = K$$

where E_2 = motor induced emf at stand still (or) start

E_1 = stator induced emf per phase

$$\boxed{\frac{E_2}{E_1} = \frac{\text{Rotor emf / phase}}{\text{Stator emf / phase}} = K = \frac{\text{Rotor turns / phase}}{\text{Stator turns / phase}}}$$

Q1) A 1000 V, 50 Hz, 3 ϕ induction motor has start connected stator. The ratio of stator to rotor turns is 3.6. The standstill impedance of rotor per phase is $0.01 + j0.02 \Omega$. Calculate

(i) Rotor current at start

(ii) Rotor power factor at start

(iii) Rotor current at a slip of 3%

Soln

Given :-

$$V_L = 1000 \text{ V}$$

$$V_{pn} = 577.35 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$\frac{\text{Stator turns}}{\text{Rotor turns}} = 3.6$$

$$K = \frac{\text{Rotor turns}}{\text{Stator turns}} = \frac{1}{3.6} = 0.277$$

$$Z_2 = R_2 + jX_2 = 0.1 + j0.2 = \text{stand still value}$$

To find :-

(i) I_2

(ii) $\cos \phi_2$

(iii) I_{2m} at $s = 3\%$

Soln

$$I_2 = \frac{E_2}{Z_2}$$

$$E_{1(L)} = 1000 \text{ V}$$

$$E_{1(pn)} = 577.35 \text{ V}$$

$$E_{2(pn)} = K \times E_1$$

$$= 0.277 \times 577.35$$

$$= \underline{\underline{159.88 \text{ V} \quad 160.375 \text{ V}}}$$

$$\rightarrow I_2 = \frac{E_2}{Z_2}$$

$$= \frac{155.88 \angle 39.99^\circ 160.375}{0.01 + j0.2}$$

$$= 7780.42 \angle -1.52^\circ$$

$$= 39.99 - j799.87$$

$$\underline{I_2 = 800.87 \angle -1.52^\circ \text{ A}}$$

$$\rightarrow \cos \phi_2 = \frac{R_2}{Z_2}$$

$$= \frac{0.01}{0.01 + j0.2}$$

$$= 0.0499 \angle -1.52^\circ$$

$$\rightarrow \beta = 0.03$$

$$I_{2M} = \frac{E_{2M}}{Z_{2M}}$$

$$= \frac{8 E_2}{\sqrt{R_2^2 + (\beta X_2)^2}}$$

$$= \frac{0.03 \times 39.99 \times 160.375}{\sqrt{(0.01)^2 + (0.03 \times 0.2)^2}}$$

$$= 412.056 \text{ A}$$

⇒ Torque equation of 3 ϕ induction motor:-

The torque produced in an IM depends on

- (i) The RMF which is responsible for inducing emf in the rotor (Φ)
- (ii) The magnitude of rotor current in running condition (I_{2r})
- (iii) The power factor of the motor in running condition ($\cos \phi_{2r}$)

Mathematically it is expressed as torque

$$T \propto \Phi I_{2r} \cos \phi_{2r}$$

The flux (Φ) or RMF Φ produced by stator winding is proportional to E_1 (stator voltage or induced emf)

WKT

E_1 and E_2 are related by the turns ratio i.e., K

$$K = \frac{\text{Rotor turns / phase}}{\text{Stator turns / phase}} = \frac{E_2}{E_1}$$

$$\therefore K = \frac{E_2}{E_1}$$

$$E_2 = K E_1$$

Assuming constant of proportionality

$$K = 1$$

$$E_2 = E_1$$

If

$$\Phi \propto E_1$$

$$E_2 = E_1$$

$$\Phi \propto E_2$$

sub in torque expression

$$T \propto E_2 I_{2m} \cos \phi_{2m}$$

$$T = K E_2 I_{2m} \cos \phi_{2m}$$

where $K =$ constant of proportionality

$$\text{for } 3-\phi \text{ IM} \Rightarrow K = \frac{3}{2\pi N_s} = \frac{3}{2\pi n_s}$$

$$\text{where } n_s = \frac{N_s}{60} \text{ rpm}$$

$$T = \frac{3}{2\pi N_s} E_2 \times \frac{E_{2m}}{Z_{2m}} \times \frac{R_2}{Z_{2m}}$$

$$T = \frac{3 E_2 \times \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}}{2\pi n_s}$$

$$T = \frac{3 s E_2^2 R_2}{2\pi n_s (R_2^2 + (sX_2)^2)} \quad \text{N-m}$$

⇒ Ex

⇒ Starting torque

at start $s = 1$

$$T = \frac{3 E_2^2 R_2}{2\pi n_s (R_2^2 + (X_2)^2)} \quad \text{N-m}$$

Q1) A 3 ϕ , 400V 50Hz, 4 pole IM has star connected stator winding. The motor resistance and reactance are 0.1 Ω and 1 Ω respectively at standstill. The full load speed is 1440 rpm calculate the torque developed on full load by the motor. Assume stator to motor ratio as 2%)

Soln :- Given :-

$$E_{1m} = 400 \text{ V}$$

$$E_{1ph} = 230.94$$

(star connected)
rpm

$$f = 50 \text{ Hz}$$

$$P = 4$$

$$R_2 = 0.1 \Omega$$

$$X_2 = 1 \Omega$$

$$N = 1440 \text{ rpm}$$

$$\frac{\text{stator turns/phs}}{\text{rotor turns/ph}} = \frac{2}{1}$$

$$K = \frac{1}{2} = 0.5$$

To find :- ?

$$T = \frac{3 s E_2^2 R_2}{2 \pi N_s (R_2^2 + (s X_2)^2)}$$

$$N_s = \frac{120 f}{P}$$

$$= \frac{120 \times 50}{4}$$

$$= 1500$$

$$N_s = 1500$$

$$n_s = \frac{N_s}{60} = \frac{1500}{60} \text{ rpm}$$

$$n_s = 25 \text{ rpm}$$

$$\mu = 2 \times 10^{-3}$$

$$\rightarrow \delta = \frac{N_s - N}{N_s}$$

$$= \frac{1500 - 1440}{1500}$$

$$= 0.04$$

$$\mu = 2 \times 10^{-3}$$

$$\frac{E_2}{E_1} = K$$

$$E_2 = 230.94 \times 0.5$$

$$= 115.47 \text{ V}$$

$$= \frac{3 \times 0.04 \times (115.47)^2 \times 0.1}{2 \times \pi \times 25 \times (10.01)^2 + (0.004 \times 1)^2}$$

$$= \underline{\underline{87.77 \text{ N-m}}}$$

verify

→ condition for maximum torque (T_m) :-

W.K.T

the torque depends on slip at which the motor is running

$$T = \frac{K s E_2^2 R_2}{(R_2^2 + (s X_2)^2)} \quad \text{N-m} \quad K = \frac{3}{2\pi N_p}$$

where K , E_2 , R_2 and X_2 are constants

To find the condition for maximum torque using maxima, minima theorem -

$$\frac{dT}{ds} = 0$$

where $T = \frac{K s E_2^2 R_2}{(R_2^2 + (s X_2)^2)} \quad \text{N-m}$

~~$$\frac{dT}{ds} = \frac{(R_2^2 + (s X_2)^2) (K E_2^2 R_2) - K s E_2^2 R_2 (2 s X_2^2)}{(R_2^2 + (s X_2)^2)^2}$$~~

Differentiate w.r.t 's'

$$\frac{dT}{ds} = \frac{(R_2^2 + (s X_2)^2) (K E_2^2 R_2) - K s E_2^2 R_2 (2 s X_2^2)}{(R_2^2 + (s X_2)^2)^2} = 0$$

$$(R_2^2 + (s X_2)^2) (K E_2^2 R_2) = K s E_2^2 R_2 (2 s X_2^2)$$

$$R_2^2 + (s X_2)^2 = 2 s^2 X_2^2$$

$$R_2^2 = 2 s^2 X_2^2 - s^2 X_2^2$$

$$R_2^2 = s^2 X_2^2$$

$$s^2 = \frac{R_2^2}{X_2^2}$$

$$s = \frac{R_2}{X_2}$$

$$\Rightarrow s_m = \frac{R_2}{X_2}$$

= slip at maximum torque

⇒ Magnitude of maximum torque :-

WKT

$$T = \frac{K s E_2^2 N_s}{R_2^2 + (s X_2)^2} \quad \text{N-m}$$

Replacing s by s_m in the above equation we get magnitude of maximum torque

~~$$T_m = \frac{K s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2}$$~~

$$T_m = \frac{K s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2} = \frac{K E_2^2}{2 X_2} = T_m$$

where $s_m = R_2 / X_2$

Q1) Calculate the torque exerted by an
 8 pole, 50 Hz, 3 ϕ - IM operating
 with a 4% slip which develops
 a maximum torque of 150 kg-m
 at a speed of 660 rpm. The resistance
 per phase of the motor is 0.5Ω

Soln:-

$$P = 8$$

$$f = 50 \text{ Hz}$$

$$s = 0.04$$

$$T = 150 \text{ kg-m} = 1470 \text{ N-m} \quad (150 \times 9.8 = 1470 \text{ N-m})$$

$$N_m = 660 \text{ rpm}$$

$$R_2 = 0.5 \Omega$$

(at maximum torque
 speed is also maximum
 Nm)

To find:-

$$T = \frac{3}{2\pi N_s} \times \frac{s E_2^2 R_2}{(R_2^2 + (sX_2)^2)}$$

$$N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{8}$$

$$= 750 \text{ rpm}$$

$$N_s = \frac{N_s}{60} = \frac{750}{60} = 12.5 \text{ rps}$$

$$\rightarrow s_m = \frac{R_2}{X_2}$$

$$\rightarrow s_m = \frac{N_s - N_m}{N_s}$$

$$= \frac{750 - 660}{750}$$

$$= 0.12$$

$$\rightarrow X_2 = \frac{0.5}{0.12}$$

$$= 4.16 \Omega$$

$$T_m = \frac{K E_2^2}{2X_2}$$

$$E_2 = \sqrt{\frac{T_m \times X_2}{K}}$$

$$= \sqrt{1470 \times 4.16}$$

$$K E_2^2 = T_m \times 2X_2$$

$$= 1470 \times 2 \times 4.16$$

$$K E_2^2 = 12242.88 \text{ V}$$

$$T = \frac{K S E^2 R_2}{R_2^2 + (S X_2)^2}$$

$$= \frac{K E^2 S R_2}{R_2^2 + (S X_2)^2}$$

$$= \frac{12242.88 \times 0.004 \times 0.5}{(0.5)^2 + (0.004 \times 4.16)^2}$$

$$T = \underline{\underline{881.76 \text{ Nm}}}$$

Q2) A 3 ϕ IM having 6 poles star connected stator winding runs on 240 V, 50 Hz supply. The motor resistance & reactance on standstill are 0.12 Ω and 0.85 Ω per phase respectively. The ratio of stator to motor turns is 1.8 and full load slip is 4%. Calculate the torque developed at full load, maximum torque and the speed at maximum torque.

Soln 

Given :-

$$E_{1(L)} = 240 = E_{1(ph)} = 138.56 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$R_2 = 0.12 \Omega$$

$$X_2 = 0.85 \Omega$$

$$\frac{\text{stator turns / phase}}{\text{rotor turns / phase}} = 1.8$$

$$K = \frac{1}{1.8} = 0.56$$

$$\delta_m = 0.04$$

To find :-
(i) T
(ii) T_m
(iii) Nm

Soln :-

$$T = \frac{K \delta E_2^2 R_2}{(R_2^2 + (\delta X_2)^2)}$$

$$E_2 = K E_1 = 0.56 \times 138.56$$
$$= 77.59$$

$$K = \frac{3}{2\pi N_s} = \frac{3}{2\pi \times 16.66} = 0.028$$

$$N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{6} = 1000$$

$$N_s = \frac{1000}{60} = 16.66$$

$$T = \frac{0.028 \times (77059)^2 \times 0.004 \times 0.12}{((0.12)^2 + (0.004 \times 0.85)^2)}$$

$$= \underline{52001 \text{ N-m}}$$

$$(ii) \rightarrow T_m = \frac{K \cdot g_m \cdot E_s^2 \cdot R_2}{R_2^2 + (g_m \cdot X_2)^2}$$

$$g_m = \frac{R_2}{X_2} = \frac{0.12}{0.85} = 0.141$$

$$= \frac{0.028 \times 0.141 \times (77059)^2 \times 0.12}{(0.12)^2 + (0.141 \times 0.85)^2}$$

$$= \underline{99.15 \text{ N-m}}$$

$$(iii) \quad N_m$$

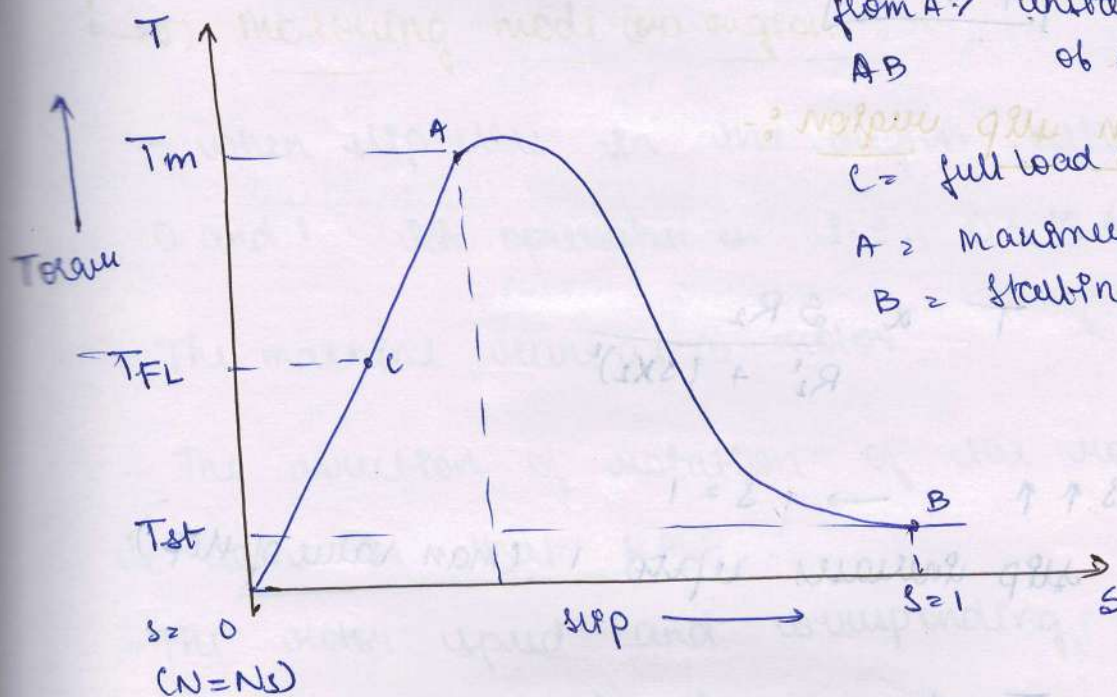
$$N = N_s (1 - s)$$

$$N_m = N_s (1 - s_m)$$

$$= 1000 (1 - 0.141)$$

$$N_m = \underline{859 \text{ rpm}}$$

→ Torque slip characteristics :-



OA = stable region

from A → unstable region
of operation

C = full load torque

A = maximum torque

B = starting torque.

↳ It consists of

- low slip region
- High slip region

Ex) Low slip region :-

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

If supply voltage V_g remains constant
then E_2 is also constant

$$T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}$$

Slip ↓ ↓ ↓ → $(s X_2)^2$ ↓ ↓

If slip is low then $(s X_2)^2$ will be

∴ Neglecting $(sX_2)^2$ term from denominator

$$T \propto \frac{sR_2}{R_2^2} \propto \frac{s}{R_2}$$

R remains constant

$$\boxed{T \propto s}$$

→ (ii) High slip region :-

WKT

$$T \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$

as $s \uparrow \uparrow \rightarrow s \geq 1$

as slip increases upto 1 (max value of slip)

$T \propto$

R_2 remains constant

$$T \propto \frac{s}{s^2 X_2^2}$$

$$\boxed{T \propto \frac{1}{s}}$$

$X_2 =$ constant at standstill

$$\boxed{T_{FL} < T_m}$$

Full load torque will be always less than maximum torque.

motoring,

⇒ Generating and braking regions of

modes of operation:-

↳ (i) motoring mode ^{normal} or region:-

- when slip lies in the region between 0 and 1 i.e., $0 \leq s \leq 1$
- The machine runs as a motor
- The direction of rotation of the motor is same as that RMF.
- The motor speed and corresponding torque is in the same direction

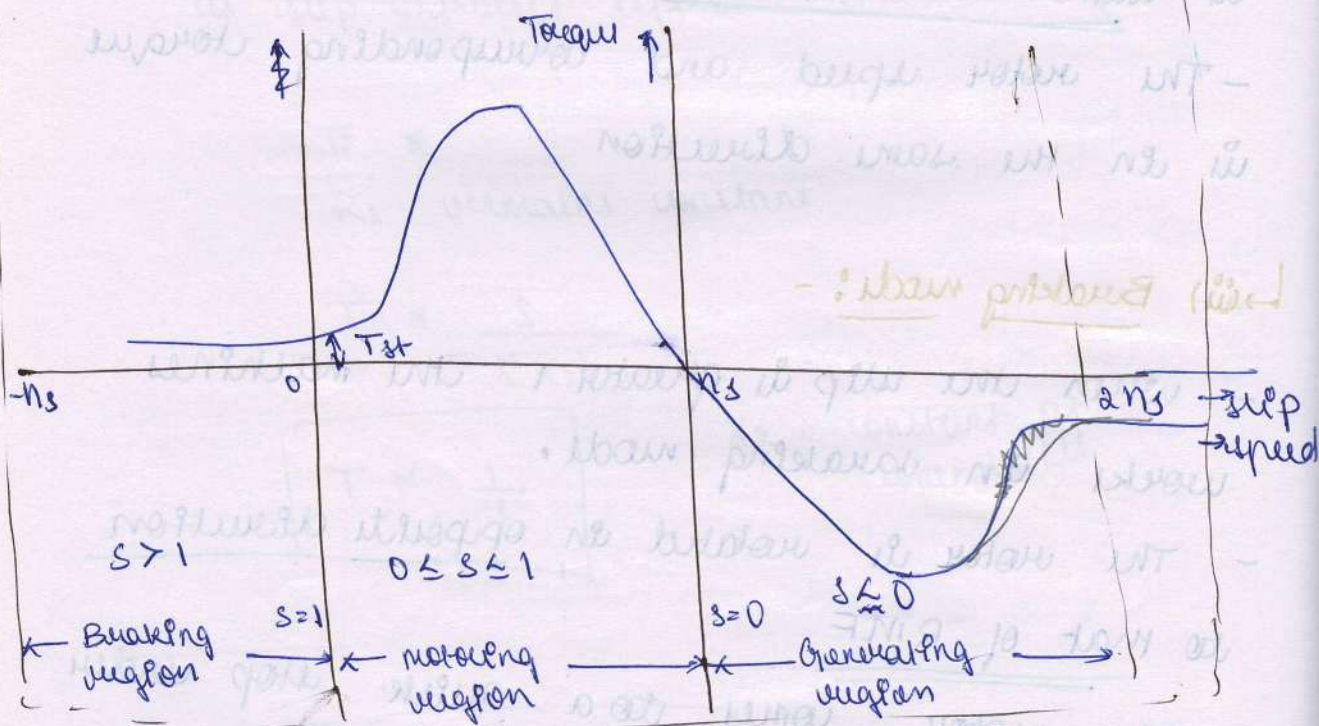
↳ (ii) Braking mode:-

- when the slip is greater 1 the machine works in braking mode.
- The motor is rotated in opposite direction to that of RMF
- The motor comes to a quick stop under the influence of counter torque, which produces braking action
- The method is also known as plugging

← reverse rotation

1. (iii) Generating mode

- The value of the slip must be less than zero, (negative slip) which indicates that the motor is rotating at a speed above synchronous speed
- The nature of torque slip characteristics reverses in generating mode



⇒ Torque variations:-

→ Torque ratios:-

Ex) Full load and max torque ratios:-

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

→ For full load $s = s_F$

$$T_{FL} \propto \frac{s_F E_2^2 R_2}{(R_2)^2 + (s_F X_2)^2}$$

→ For maximum load $s = s_m$

$$T_m \propto \frac{s_m E_2^2 R_2}{(R_2)^2 + (s_m X_2)^2}$$

$$\therefore \frac{T_{FL}}{T_m} = \frac{s_F E_2^2 R_2}{R_2^2 + (s_F X_2)^2} \times \frac{R_2^2 + (s_m X_2)^2}{s_m E_2^2 R_2}$$

$$\frac{T_{FL}}{T_m} \propto \frac{s_F}{s_m} \times \frac{(R_2^2 + (s_m X_2)^2)}{(R_2^2 + (s_F X_2)^2)}$$

Dividing both Num & Den by X_2^2

$$\frac{T_{FL}}{T_m} \propto \frac{s_F (R_2^2 / X_2^2 + s_m^2)}{s_m (R_2^2 / X_2^2 + s_F^2)}$$

$$W.K.T \quad = \frac{R_2}{X_2} = s_m$$

$$\frac{R_2^2}{X_2^2} = s_m^2$$

$$\frac{T_{FL}}{T_m} \propto \frac{s_F}{s_m} \times \frac{(s_m^2 + s_F^2)}{(s_m^2 + s_F^2)}$$

$$\propto \frac{s_F}{s_m} \times \frac{2s_m}{(s_m^2 + s_F^2)}$$

$$\frac{T_{FL}}{T_m} \propto \frac{2s_F s_m}{s_m^2 + s_F^2}$$

$$\frac{T_{FL}}{T_m} = K \frac{2s_F s_m}{s_m^2 + s_F^2}$$

$$\text{If } K = 1$$

$$\boxed{\frac{T_{FL}}{T_m} = \frac{2s_F s_m}{s_m^2 + s_F^2}}$$

Ex 10.1) Starting torque and maximum torque
values:-

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \rightarrow (1)$$

At start

$$s = 1$$

$$T_{st} \propto \frac{E_2^2 R_2}{R_2^2 + X_2^2} \rightarrow (2)$$

$$\text{If } s = s_m$$

then

$$T_m \propto \frac{s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2} \rightarrow (3)$$

Dividing ② and ③

$$\frac{T_{st}}{T_m} = \frac{E_2^2 R_2}{(R_2^2 + X_2^2)} \times \frac{(R_2^2 + (s_m X_2)^2)}{s_m E_2^2 R_2}$$

$$\frac{T_{st}}{T_m} = \frac{R_2^2 + s_m^2 X_2^2}{s_m [R_2^2 + X_2^2]}$$

Dividing both Num and Den by X_2^2

$$= \frac{\frac{R_2^2}{X_2^2} + s_m^2}{s_m \left[\frac{R_2^2}{X_2^2} + 1 \right]}$$

Substitute $R_2/X_2 = s_m$

$$\frac{T_{st}}{T_m} = \frac{s_m^2 + s_m^2}{s_m (s_m^2 + 1)}$$

$$= \frac{2s_m^2}{s_m (1 + s_m^2)}$$

$$\boxed{\frac{T_{st}}{T_m} = \frac{2s_m}{1 + s_m^2}}$$

Q1) A 3 phase, 50 Hz IM runs on full load with a slip of 4%. The motor standstill impedance per phase $0.0 + j0.05 \Omega$. Calculate the available maximum torque in terms of full load torque. Also determine speed at which maximum torque occurs.

Soln

Given :-

$$s\% = 4\%$$

$$s = 0.04 = s_f = 0.04$$

$$P = 6$$

$$f = 50 \text{ Hz}$$

$$Z_2 = (0.001 + j0.005) \Omega/\text{ph}$$

$$R_2 = 0.001 \Omega/\text{ph}$$

$$X_2 = 0.005 \Omega/\text{ph}$$

To find :-

T_m and N_m of T_{st}
and N_m at T_m

Soln

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (X_2 s)^2}$$

R E_2 remains constant
if the supply voltage
remains constant

$$T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}$$

$$\rightarrow \frac{T}{T_{FL}} \propto \frac{s_F R_2}{R_2^2 + (s_F X_2)^2}$$

$$\rightarrow T_m \propto \frac{s_m R_2}{R_2^2 + (s_m X_2)^2}$$

$$I_m = \frac{R_2}{X_2}$$

$$= \frac{0.001}{0.005} = 0.2$$

$$\rightarrow T_m = \frac{(0.2)(0.001)}{(0.001)^2 + (0.2)^2(0.005)^2} \quad [K=1]$$

$$T_m = 10$$

$$\rightarrow T_{FL} = \frac{(0.004)(0.001)}{(0.001)^2 + (0.004 \times 0.005)^2}$$

$$T_{FL} = 3.84$$

$$\frac{T_{FL}}{T_m} = \frac{3.84}{10} = 0.384$$

$$T_m = \frac{T_{FL}}{0.384} = 2.6 T_{FL}$$

$$T_m = 2.6 T_{FL}$$

$$\rightarrow N_s = N_d(1-s)$$

$$\rightarrow N = N_m(1-s_m)$$

$$N_m$$

$$1-s_m$$

$$\rightarrow N = N_s (1 - s)$$

at max

$$\rightarrow N_m = N_s (1 - s_m)$$

$$N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{6} = 1000$$

$$N_m = 1000 (1 - 0.2)$$

$$\rightarrow N_m = 800 \text{ rpm}$$

$$\underline{N_m = 800 \text{ rpm}}$$

Q2) A 746 kW, 3 ϕ , 50 Hz, 16 Pole IM

has a motor impedance of $0.02 + j0.15 \Omega$

at standstill. The full load torque

is obtained at 360 rpm. Calculate

(i) speed at which maximum torque

occurs

(ii) Ratio of maximum to full load torque

(iii) The external resistance / ph to be inserted in the motor circuit to get maximum torque at start

Soln

$$P = 16$$

$$f = 50 \text{ Hz}$$

$$Z_2 = 0.02 + j0.15$$

$$P_{out} = 746 \text{ kW} = 0.1 P \text{ power}$$

$$R_2 = 0.02 \text{ } \Omega/\text{ph}$$

$$X_2 = 0.15 \text{ } \Omega/\text{ph}$$

$$N_{m2} = 3600 \text{ rpm}$$

Soln To find :-

$$(i) N_m = ?$$

$$(ii) T_m / T_{fl} = ?$$

$$(iii) R_{m}/\text{ph} = ?$$

Soln

(i)

$$N_m = N_s (1 - s_m)$$

$$N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{16}$$

$$= 375 \text{ rpm}$$

$$\rightarrow s_m = \frac{R_2}{X_2} = \frac{0.02}{0.15} = 0.133$$

$$N_m = 375 (1 - 0.133)$$

$$= 325 \text{ rpm}$$

(ii)

$$\rightarrow T_m \propto \frac{S_m R_2}{R_1^2 + (S_m X_2)^2}$$

$$= \frac{(0.0133)(0.002)}{(0.002)^2 + ((0.0133)(0.015))^2}$$

$$T_m = 3.33$$

→

~~To find~~

$$S = \frac{N - N_s}{N_s}$$

$$360 = 375 - 3$$

$$\rightarrow \text{To find } S_f \Rightarrow N = N_s(1 - S_f)$$

$$360 = 375(1 - S_f)$$

$$S_f = 0.04$$

WKT

~~T_m~~

$$T_{S_f} = \frac{T_m + 2S_m S_f}{S_m^2 + S_f^2}$$

$$= \frac{3.33 + 2 \times 0.133 \times 0.04}{(0.133)^2 + (0.04)^2}$$

$$= \underline{\underline{1.812}}$$

(iii)

for T_m at start $s_m \approx 1 = \frac{R_2}{X_2}$

∴ $R_2 \approx X_2$

$R_2 \approx X_2 \approx 0.15 \Omega \rightarrow$ (new value of reactance)

Run ~~at~~ that needs to be added

is

$R_{en} = 0.15 - 0.02 = 0.13 \Omega / \text{ph}$
(0.02 = old value of resistance)

Q3) A 4-pole 3ϕ , 50 Hz IM develops a maximum torque of 30 Nm at 960 rpm. Determine the torque exerted by the motor at 5% slip. The motor resistance per phase is 0.6Ω .

Soln

Given

$p = 4$

$f = 50 \text{ Hz}$

$T_m = 30 \text{ Nm}$

$N_m = 960$

$R_2 = 0.6 \Omega / \text{ph}$

$s = 0.05$

To find

torque $T = ?$

8

$$\rightarrow N_m = N_s (1 - s_m)$$

(iii)

$$s_m = \frac{R_2}{X_2}$$

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$960 = 1000(1 - s_m)$$

$$0.96 = 1 - s_m$$

$$s_m = 0.04$$

$$T_m = \frac{s_m R_2}{R_2^2 + (s_m X_2)^2}$$

$$s_m = \frac{R_2}{X_2}$$

$$X_2 = \frac{0.6}{0.05} = 12 \Omega$$

$$T \propto \frac{s E^2 R_2}{R_2^2 + (s X_2)^2}$$

$E_2 = \text{constant}$

$$T \propto \frac{s R_2}{R_2^2 + (s X_2)^2} \quad (K=1)$$

$$= \frac{0.05 \times 0.6}{(0.6)^2 + (0.05 \times 12)^2}$$

$$T = 0.0325 \text{ Nm}$$

$$\frac{T_m}{T} = \frac{30}{T}$$

$$\geq \frac{30}{8R_2} \times \frac{(R_2^2 + (sX_2)^2)}{1}$$

$$\frac{T_m}{T} = \frac{30}{(0.05 \times 6)} \times (0.6)^2 + (0.05 \times 15)^2$$

→ [using (T_m)]

$$\rightarrow \frac{T_m}{T} = \frac{8mR_2}{R_2^2 + (smX_2)^2} \times \frac{R_2^2 + (sX_2)^2}{8R_2}$$

$$\frac{30}{T} = \frac{0.04}{0.05} \times \left[\frac{(0.6)^2 + (0.05 \times 15)^2}{(0.6)^2 + (0.04 \times 15)^2} \right]$$

$$T = \underline{\underline{29.26 \text{ Nm}}}$$

MODULE - 4

SINGLE PHASE

INDUCTION MOTORS

Constitution

- 1) Stator
- 2) Rotor $\rightarrow$ squirrel cage (or) short-circuited rotor

3ϕ IM	1ϕ IM
Self starting	1) Not self starting
Flux produced in rotating flux (or) field)	2) Flux produced in alternating (or) field)

Ring $\rightarrow$ state $\rightarrow$ action $\rightarrow$ next state $\rightarrow$ env $\rightarrow$ current phone $\rightarrow$

from its pre-state & total gain interest to

→ To understand why single phase induction motors are not self starting,

let us consider Double revolving field theory

* According to this theory, any alternating quantity (referring to flux) can be resolved into two rotating components which rotate in opposite directions and each having magnitude half of the maximum magnitude of the alternating quantity.

- In case of single phase induction motors the stator winding produces an alternating magnetic field say Φ_s

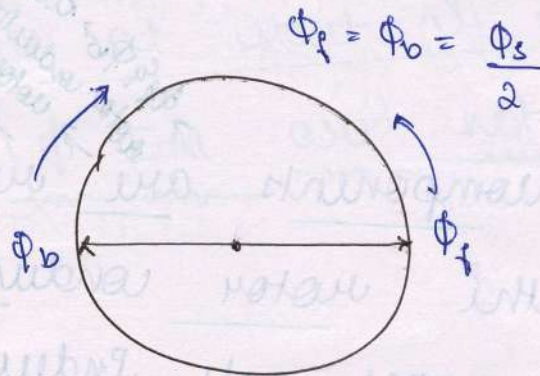
- According to double revolving theory consider the two rotating components of the stator flux say Φ_f and Φ_b

where $\Phi_f = \frac{\Phi_s}{2} = \Phi_b$

(or) $\Phi_f = \Phi_b = \frac{\Phi_s}{2}$

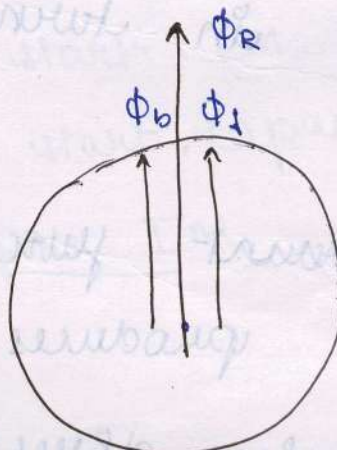
- Φ_f is the forward component rotating in anticlockwise direction and
- Φ_b is the backward component rotating in clockwise direction
- The resultant of these two components gives the originating stator flux

At start :-



$\Phi_R = 0$

After $\theta = 90^\circ$



- (i) At start both the components are opposite to each other

Thus $\Phi_R = 0$

- (ii) After $\theta = 90^\circ$ both the components are pointing in the same direction hence Φ_R is the algebraic sum of two components

- Continuous rotation of these two components gives the original alternating stator sum

- Both the components are rotating and cut the rotor conductor

and hence emf gets induced in the rotor which circulates rotor

current which in turn produces rotor sum

- This rotor current sum interacts with Φ_f and produces "torque" in anticlockwise direction and

def Φ_f cuts
rotor conductor with
rotor current
rotor sum is produced

$R(\text{amp})$
 $\downarrow$
 $R(\frac{1}{2})$
 $\downarrow$
 $R(\frac{1}{2})$

rotor sum
 $\downarrow$
 Φ_f
and rotor sum
 $\downarrow$

the same motor sum interacts with ϕ_b and produces "torque in clockwise direction" (say - ve torque)

- At start these two torques are also in opposite direction with equal magnitudes $\rightarrow$ Each torque tries to rotate the motor in its own respective direction

"The net ~~the~~ torque experienced or resultant torque experienced by the motor at start is zero!"

Hence 1 ϕ IM are not self starting.

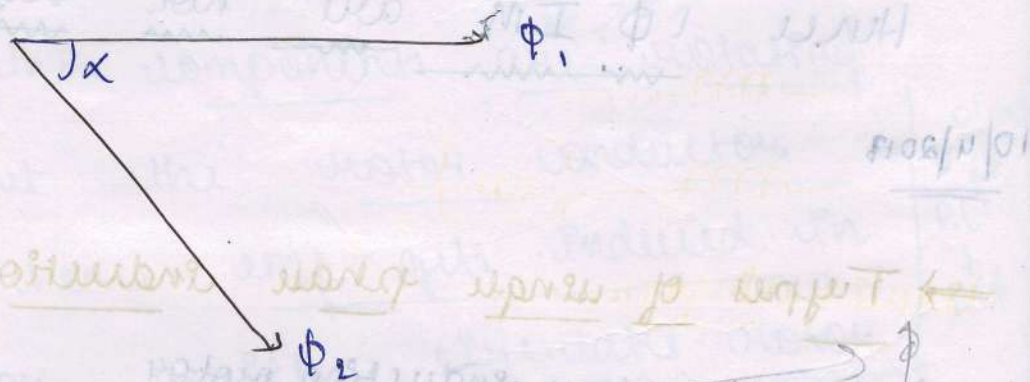
$\Rightarrow$ Types of single phase induction motor:-

- 1) Split phase induction motor
- 2) Capacitor start-IM
- 3) Capacitor start, capacitor run IM
- 4) Shaded pole IM

winding $\rightarrow$ Stator/main $\rightarrow$ Starting $\rightarrow$ Running
 Auxiliary/starting $\rightarrow$ Running
 (b) Split phase induction motor

To produce RMF, it is necessary to have minimum two alternating fluxes having a phase difference, the two.

The interaction of these two fluxes produce a resultant flux which is rotating in space at a speed of synchronous speed ($N_s = 120 f / P$)



α = split phase angle
 now the angle α , more is the starting torque produced

rotor in 1ϕ IM rotates in same direction as that of R.M.F

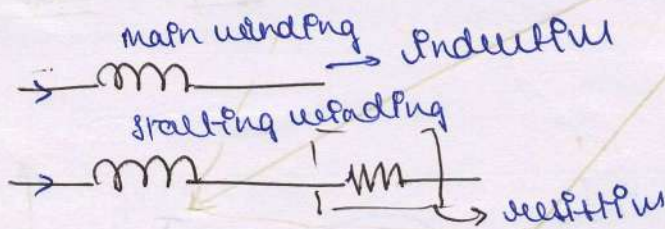
b) Split phase induction motor -

This type of motor has single phase stator winding called the main winding.

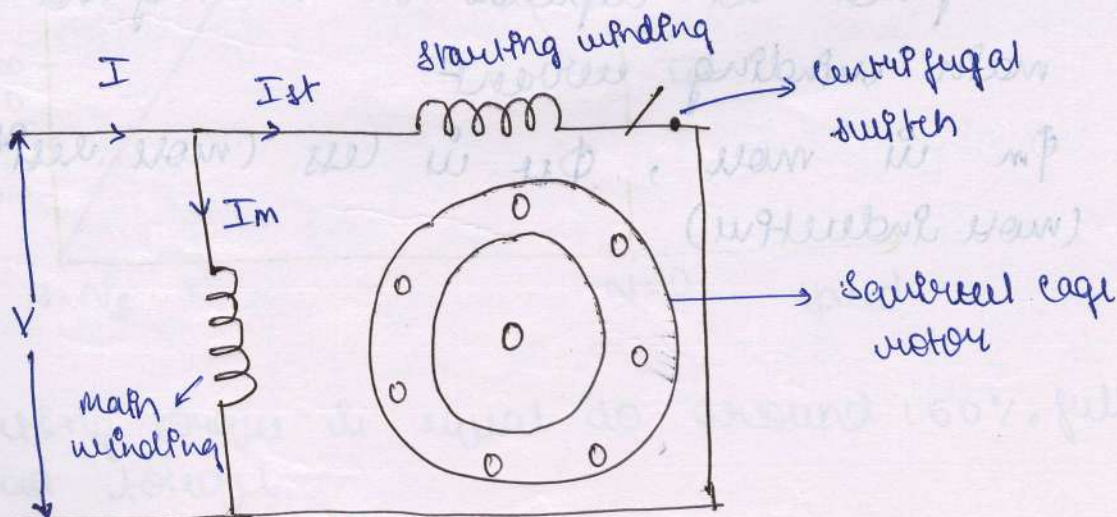
In addition to this stator carries one more winding called auxiliary winding or starting winding.

Stator winding $\rightarrow$ (or) main winding $\rightarrow$
more inductive in nature

Auxiliary winding $\rightarrow$ (or) starting winding $\rightarrow$
more resistive in nature



Circuit diagram



Let

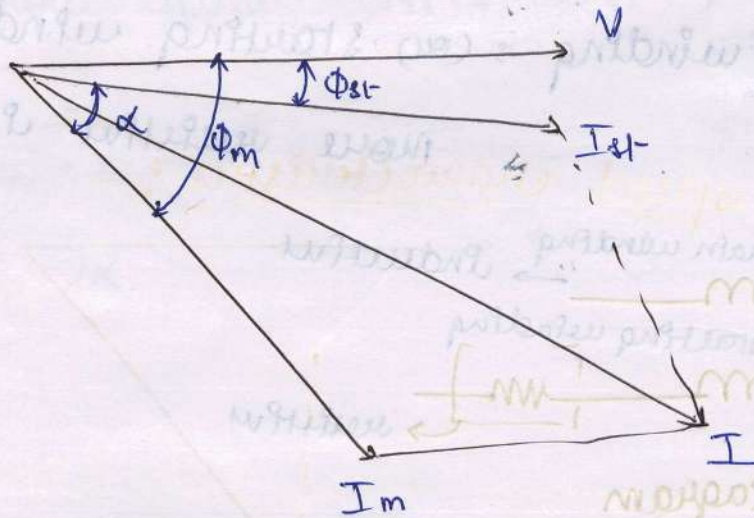
I_m = current through the main winding

I_{st} = current through the starting winding

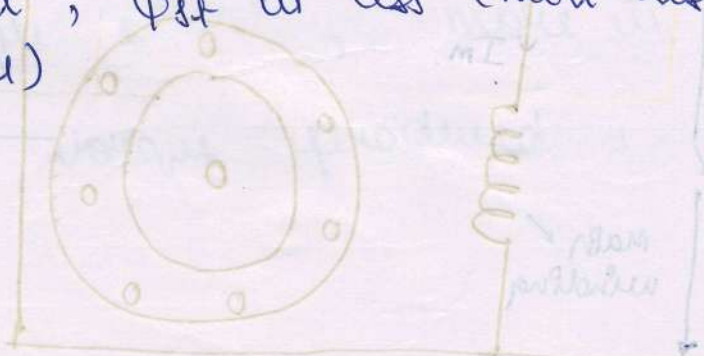
V = ~~Main~~ supply voltage

When the speed of rotor reaches 75-80% of synchronous speed centrifugal switch opens
To start the rotor RMF is required.

Phasor diagram :-



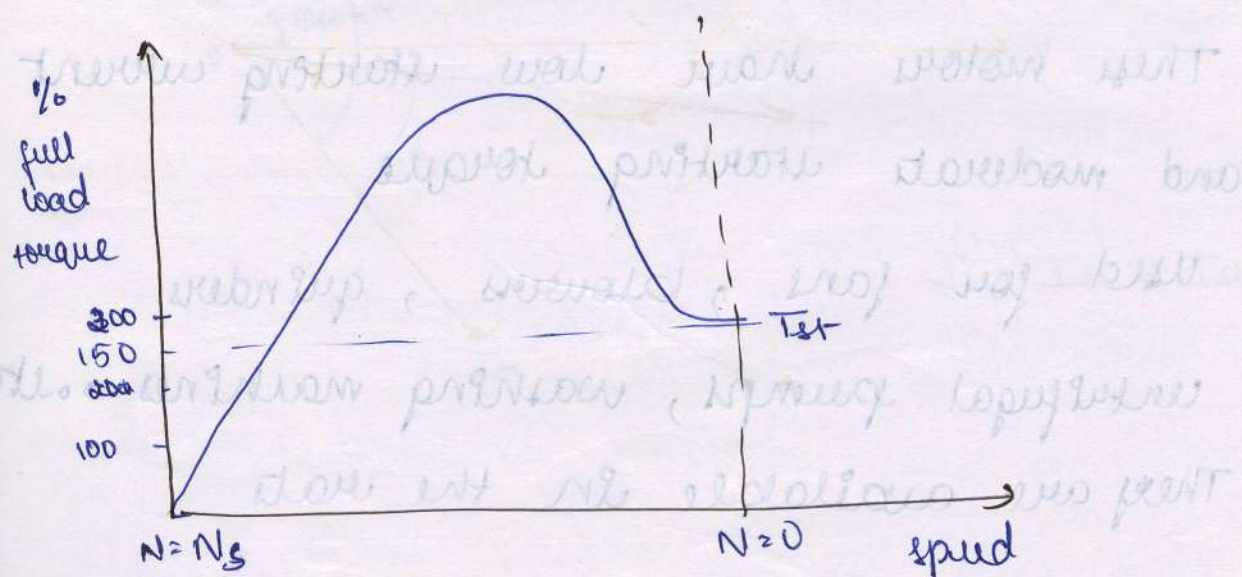
α is fixed it depends on starting and main winding current
 ϕ_m is more, ϕ_{st} is less (more self flux)
(more inductive)



Starting winding ~~is~~ has a centrifugal switch in series with it ~~when~~ when the motor attains the speed of 75 - 80% of synchronous speed, the centrifugal switch gets opened mechanically and then in running condition the starting winding gets removed ~~from~~ ^{from} the circuit.

As current I_m and I_{st} are splitted from each other by an angle α at start, the motor is called split-phase motor.

Torque speed characteristics :-



starting torque is equal to around 150% full load torque

$$T_{st} \propto \alpha$$

α = split phase angle or phase difference angle

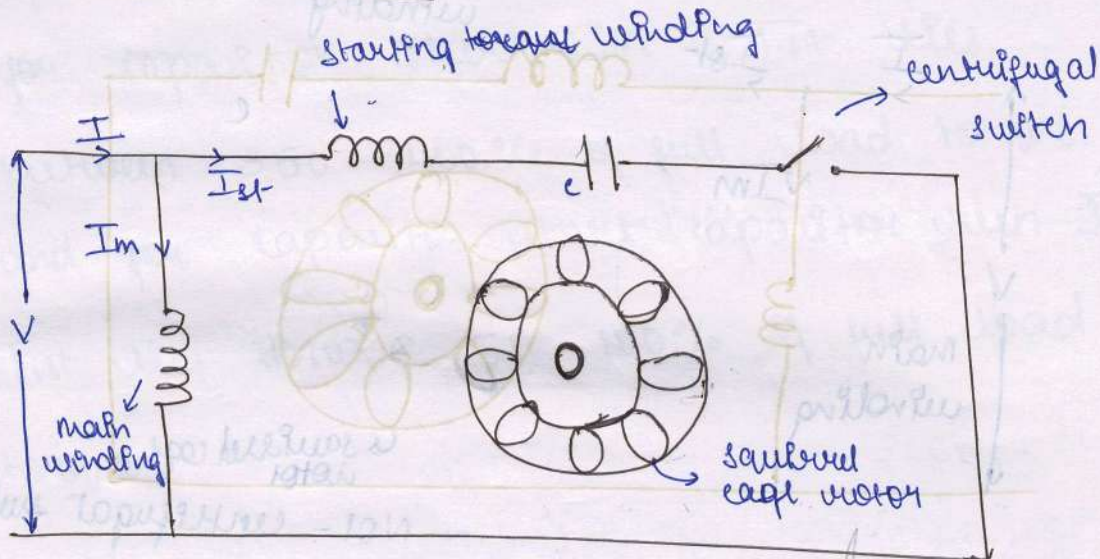
- Split phase IM have poor starting torque which is 125-150% of full load torque
- Direction of rotation of the motor can be reversed by reversing the terminals of either main winding or auxiliary winding

Applications

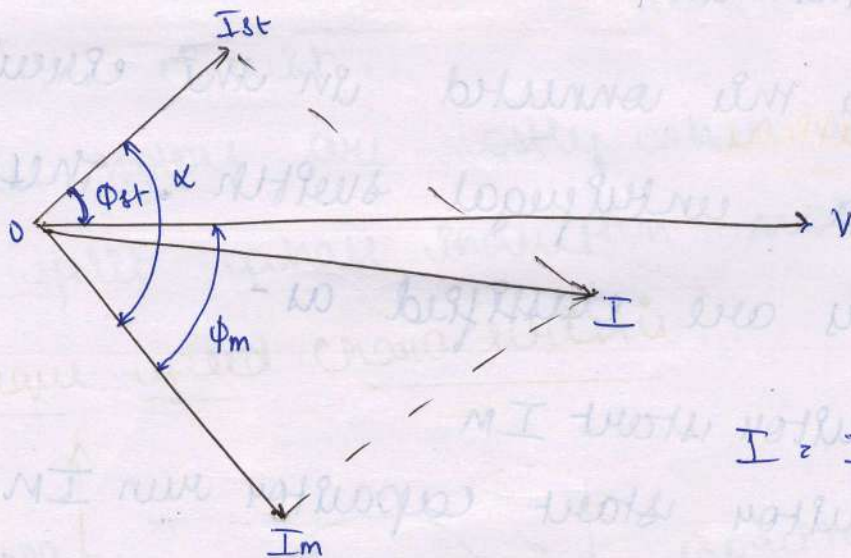
- These motors have low starting current and moderate starting torque
- used for fans, blowers, grinders, centrifugal pumps, washing machines etc.
- They are available in the range of $\frac{1}{20}$ to $\frac{1}{2}$ kW

→ iii) Capacitor start induction motor :-

→ Circuit diagram

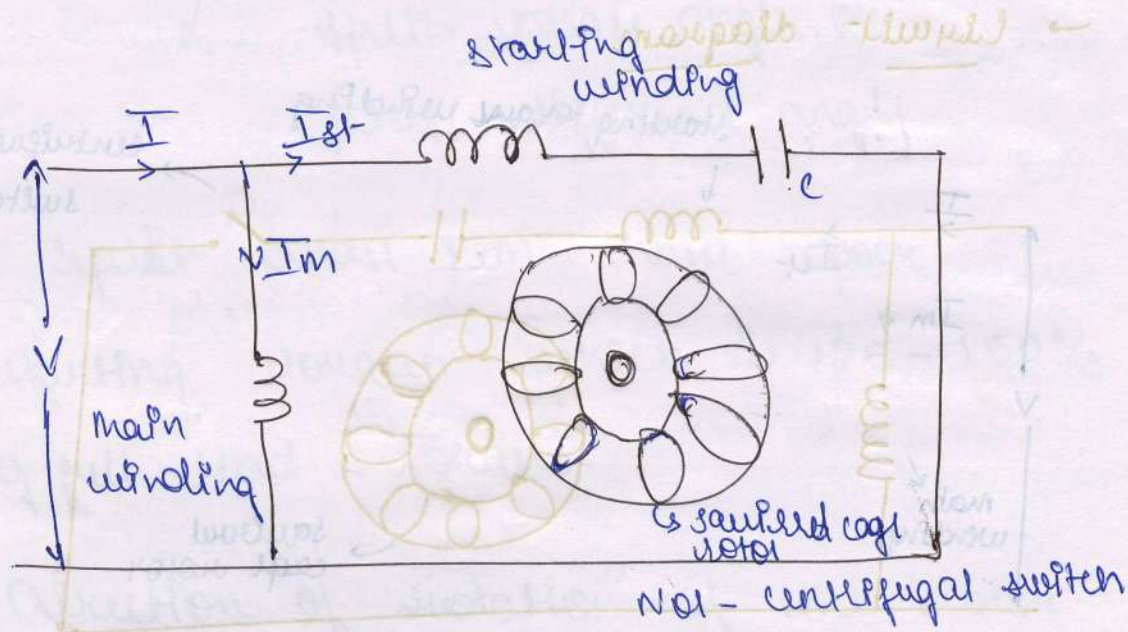


→ Phasor diagram :-



$$I = I_m + I_{st}$$

→ (iii) Capacitor start capacitor run
induction motor -



→ Depending whether the capacitor remains in the circuit permanently or is this connected in the circuit using a centrifugal switch. These motors are classified as -

(i) capacitor start IM

(ii) capacitor start capacitor run IM

Due to the capacitor the current I_{st} leads the voltage by an angle Φ_{st} hence there exists a large phase difference b/w I_m and I_{st} which is

W.K.T

- $T_{st} \propto (X)$

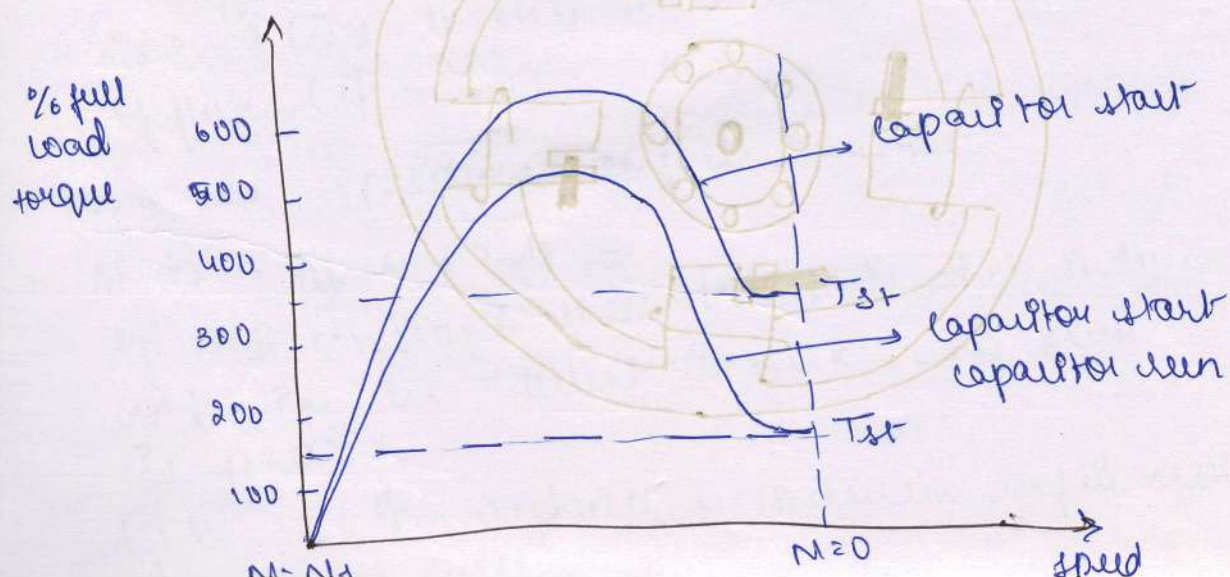
Hence such motors produce a very high starting torque i.e.,

for capacitor start IM - T_{st} lies between 350 - 400% of full load torque and for capacitor start capacitor run IM T_{st} lies below 50 - 100% of full load torque.

In case of capacitor start capacitor run IM there is no centrifugal switch and the capacitor remains permanently in the circuit.

These motors are costly when compared to split phase induction motors

→ Torque speed characteristics

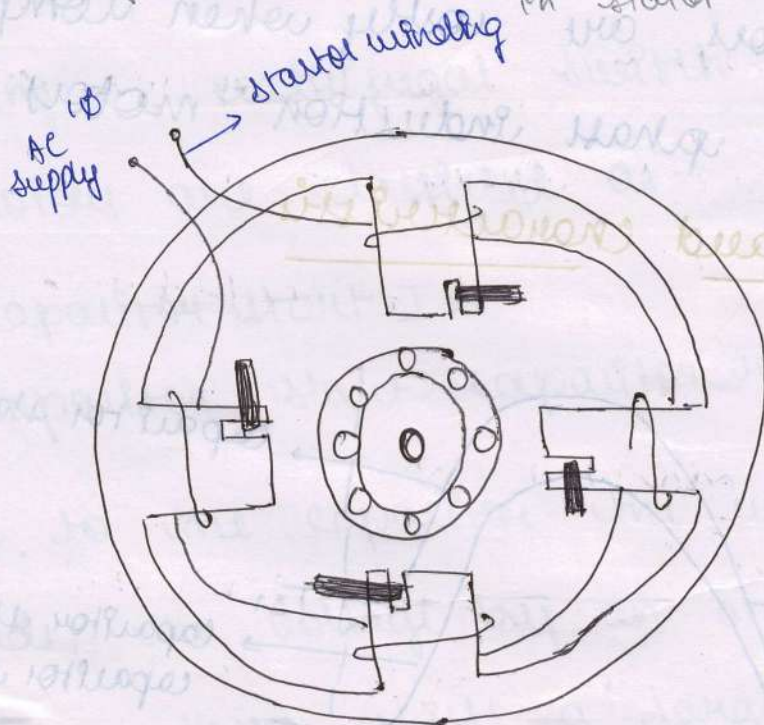


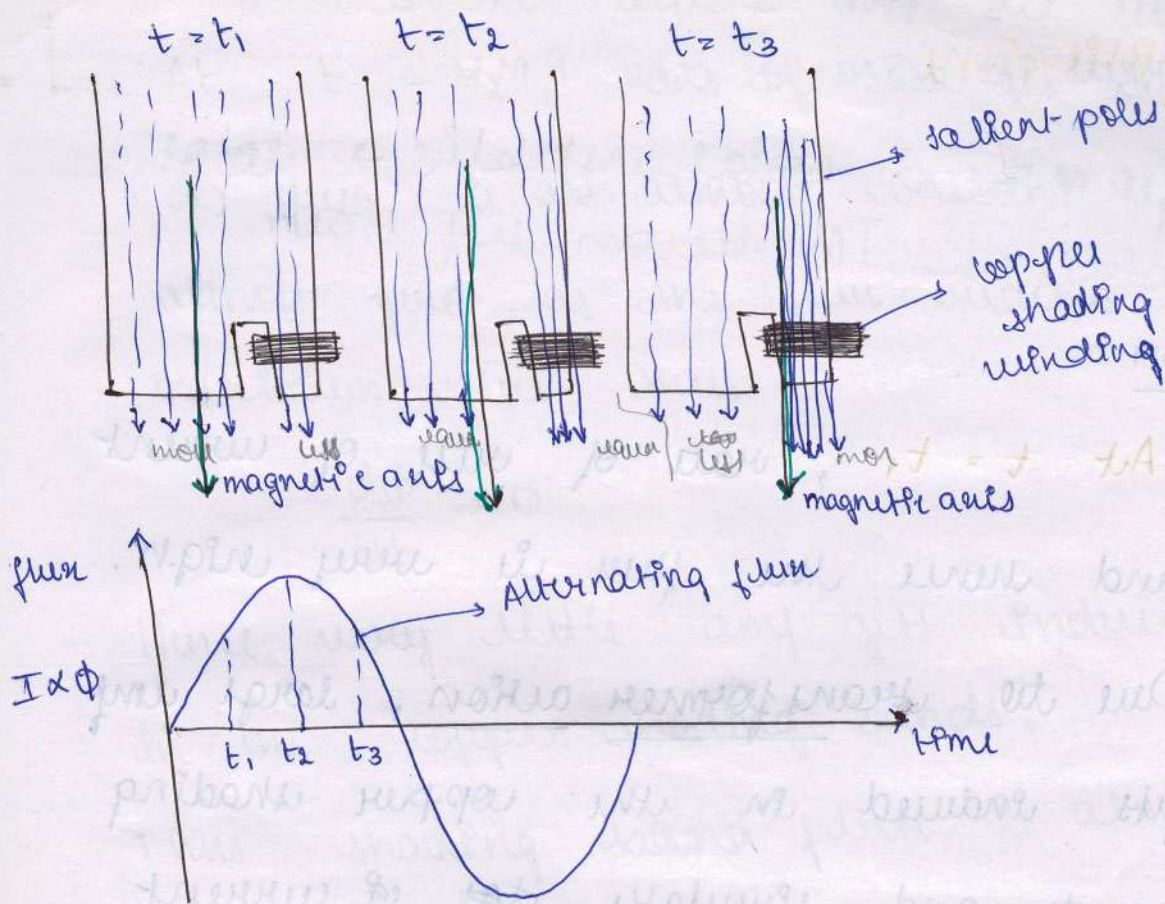
Applications

- These motors have high starting torque
- { Used for compressors, conveyors, grinders, fans, refrigerators, air conditioners etc..
→ capacitor start
- { Used for ceiling fans, blowers, air ventilators - ... etc..
→ capacitor start-capacitor run
- These motors are available upto 6 kW

→ ii) Shaded pole induction motor:-

(only this consists of physical poles)
in stator





Stator - has salient poles
 copper shading bands are mounted on these poles
 ↳ convert alternating flux to RMF

As I flow $\rightarrow \phi$ flow $\rightarrow$ when ϕ cuts Cu shading bands
 $\rightarrow$ emf gets induced in Cu shading $\rightarrow$ current starts
 flowing in Cu shading band $\rightarrow$ of produced
 ϕ of this $\rightarrow$ this Cu produced ϕ opposes main
 ϕ (Lenz's law) (ϕ_{ind})
 $\rightarrow$ Due to this there will be subtraction
 of flux ($\phi - \phi_{ind}$)
 when Cu $\rightarrow$ produced emf is maximum

At $t_2 \rightarrow \phi_{in}$ constant $\rightarrow$
 If ϕ is constant then there is no induced
 emf in Cu (current is zero) $\rightarrow$ no flux
 is prod

$t = t_3 \rightarrow \phi_{in}$ reduces $\rightarrow$ induced emf is not

due to this severe alternation
flux is converted to RMF.

If RMF is ~~total~~ established $\rightarrow$ rotor
rotates and IM begins self start

15/4/2017

At $t = t_1$, rate of rise of current

and hence the flux is very high.

- Due to transformer action, large emf
gets induced in the copper shading
bands and circulates ~~to~~ a current
through the copper shading bands
producing its own flux.
- By Lenz's law, the shading band
flux opposes main ^{stator} flux.
- Hence there is crowding / gathering /
accumulation of flux under the non
shaded part while there is weakening
of flux under the shaded part.
- Overall magnetic axis shifts to the
non shaded part shown.

- $\rightarrow$ At $t = t_2$, rate of rise of current (or) flux is ~~at~~ almost constant i.e., almost zero as the flux reaches maximum value thus

$$\frac{d\phi}{dt} = 0$$

- Hence very little emf gets induced in the copper shading bands.

Thus shading band flux is almost negligible.

- Thus it does not affect the distribution of main stator flux, ~~h~~ hence ~~at~~ the main stator flux gets ~~a~~ equally under both shaded and non shaded part of pole

- $\rightarrow$ At $t = t_3$, the current ~~and~~ ^(or) flux is reducing, the rate of decrease is high and thus a large emf gets induced in the copper shading band.

- This shading band flux is in the same

- Thus there is more flux under the shaded part when compared to non shaded part.

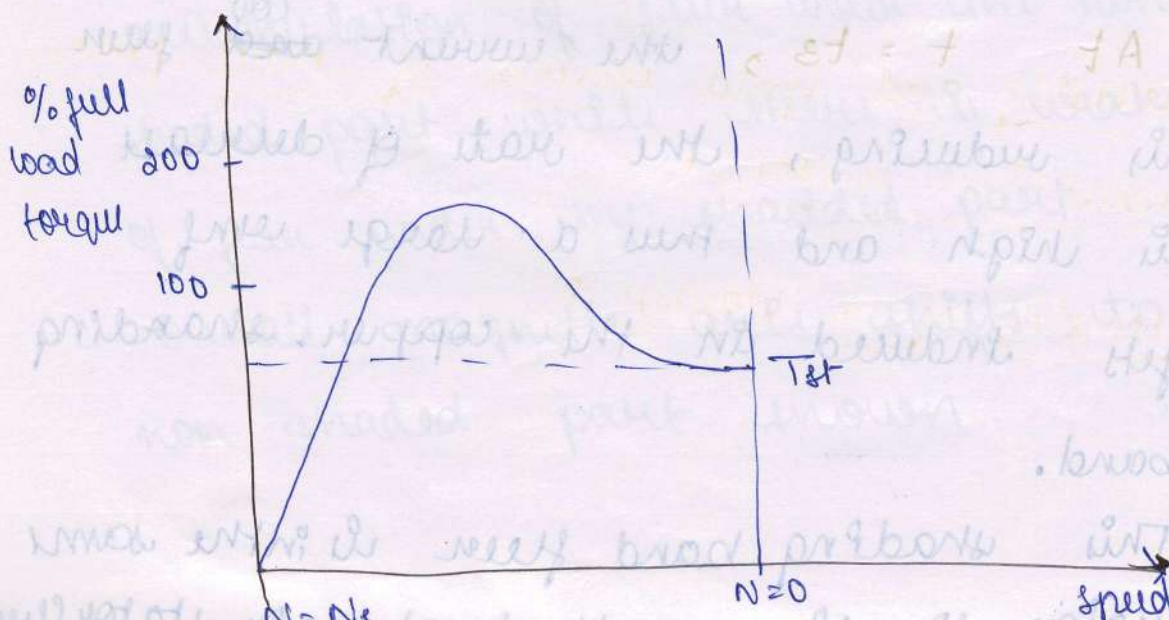
- Thus magnetic axis lies under the shaded part of the pole.

→ During -ve half cycle

This sequence keeps on repeating for -ve half cycle also and produces the effect of rotating magnetic field which is from the non shaded part of the pole to shaded part of the pole.

[+ve → shaded to non shaded part]

⇒ Torque speed characteristics:-



- starting torque is low when is about 40-50% of % full load torque

→ Limitations :-

- Starting torque is low.
- Power factor is also low.
- Due to I^2R losses (copper losses) in the shading band, the efficiency is very low
- Speed reversal is very difficult
- Size and power rating of these motor is very small. These motor are available in the range of 1/300 to 1/20 kW
- Application :-

- ~~Used~~ Used for small fans, toy motors, advertising display, film projectors, hair driers, photo copy machines, etc.
- Available in the range of $\frac{1}{300}$ to $\frac{1}{20}$ kW

⇒ Starting and speed control of three phase induction motor :-

- Necessity of a starter :-

WKT

$$I_{2u} = \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{Amps}$$

I_{2u} = motor current under running condition

But at start $s = 1$, $N = 0$

$$I_{2u} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

Thus the magnitude of motor induced emf is very large at start

$$E_{2u} = s E_2$$

$$E_{2u} = E_2 \quad [s = 1]$$

As the motor conductors are short circuited, large emf gets induced in the motor which circulates very high current in the motor at start.

- When the motor current is high, consequently the stator draws a very high current from the supplies. This current can be of the order of

~~5-10 times~~ 5-8 times the full load current at start.

3φ
AC
supply

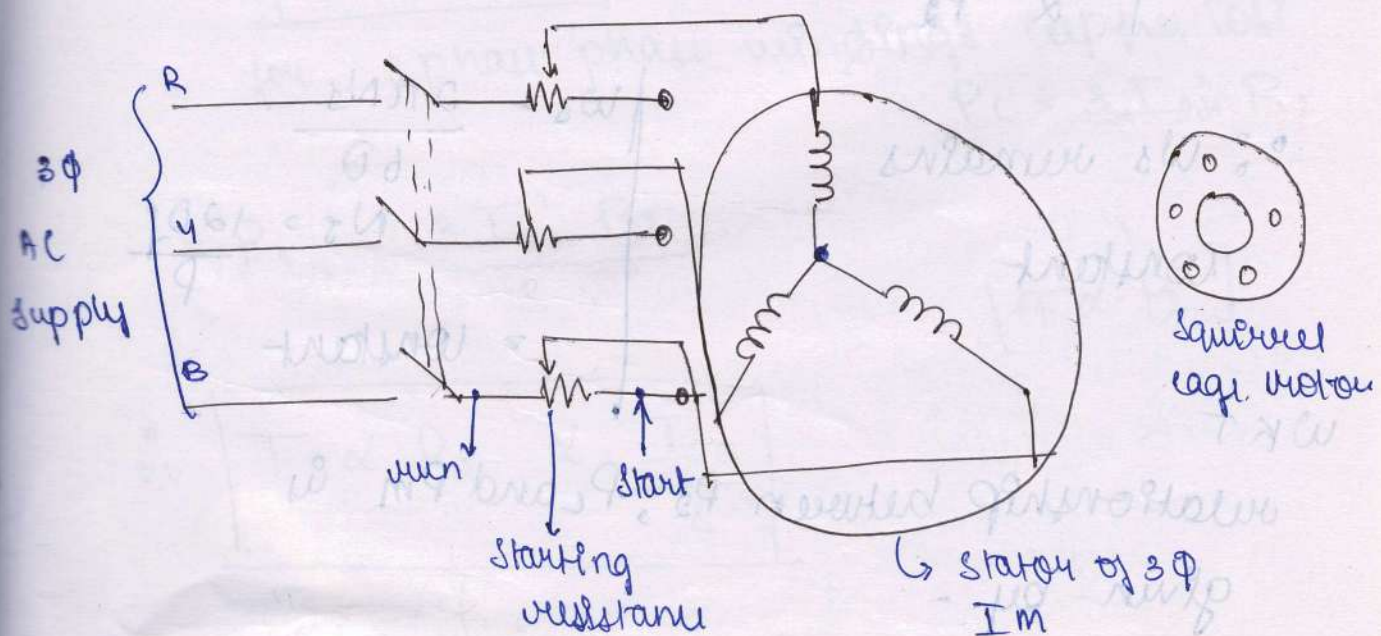
- rotor winding terminals are available only at slipping rotor
- not and not for induction motor (as internal resistance cannot be added)

- To limit the high starting current, a starter is used for a three phase induction motor.

- Types of starters used for three phase induction motors:-

- 1) Stator resistance starter
- 2) Auto-transformer starter
- 3) Star delta starter
- 4) Rotor resistance starter
- 5) Direct online (DOL) starter

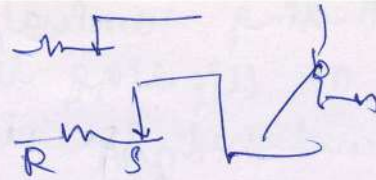
→ 1) Stator resistance starter :-



arrow at start position

(i)

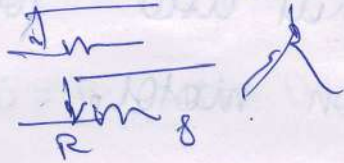
At start
resistance in
circuit (max)
so less current
flows at start



→ supply volt. → in stator reduced

P_{in} total induced
→ induced

(ii) In run
arrow at run position
resistance is
bypassed



when the motor picks up 20-80% of speed arrow comes to run position
[$I \downarrow \rightarrow$ induced emf $\downarrow \rightarrow$ flux $\downarrow \rightarrow$ motor will & amp]

→ Relationship between starting torque T_{st}
and full load torque T_{FL} :-

$$P_2 = T \times \omega_s$$

P_2 = Rotor input power at N_s

T = torque produced

$$T \propto P_2$$

∴ ω_s remains constant

$$\omega_s = \frac{2\pi N_s}{60}$$

$$N_s = \frac{120f}{P}$$

= constant

WKT

relationship between P_2 , P_c and P_m is
given by -

$$P_2 = T \times \omega_s$$

$$T \propto P_2$$

$$P_2 : P_e : P_m \text{ in } 1 : s : 1-s$$

$$\left[\frac{P_2}{P_e} = \frac{1}{s} ; \frac{P_e}{P_m} = \frac{s}{1-s} \right]$$

where P_2 = motor P/P power at Ns in watts

P_e = motor copper loss in watts

P_m = ~~gross~~ Gross mechanical power developed in the motor

$$\left[\begin{array}{l} \text{electrical equivalent of gross} \\ \text{mechanical power developed} \\ \hline \approx P_m = E_b I_a \end{array} \right]$$

$$\therefore \frac{P_2}{P_e} = \frac{1}{s}$$

$$\boxed{P_2 = \frac{P_e}{s}}$$

for 3 phase phase windings copper loss

$$P_e = 3 I_{a1}^2 R_2$$

$$P_2 = \frac{3 I_{a1}^2 R_2}{s}$$

$$[T \propto P_2]$$

$$\therefore \boxed{T \propto P_2 \propto \frac{I_{a1}^2}{s}}$$

Slip is not small
Approx
slip is not small
Rotor cu
DDL +



Note :-

The motor current I_{2M} and the stator current are related to each other through transformer action.

$$T \propto \frac{I_1^2}{s} \rightarrow \textcircled{1}$$

where I_1^2 is stator current-

$$s \approx 1, I_1 \approx I_{st} = n I_{sc}$$

$$I_{2M}^2 \propto I_1^2$$

→ At start $s = 1$, $T = T_{st}$ and $I_1 = I_{st}$

The stator voltage reduces by a factor 'n' which is always less than 1

$$\text{i.e., } \boxed{n < 1}$$

and starting current proportional to this factor 'n'

If I_{sc} is the normal current drawn under full rated voltage condition at start then

$$\boxed{I_{st} = n I_{sc}}$$

Note

∴ $\text{eqn (1)} \Rightarrow \text{becomes}$

$$T_{st} \propto \frac{(I_{st})^2}{1} \propto \left(\frac{n I_{sc}}{1} \right)^2 \quad \rightarrow (2)$$

$s = 1$

↳ Under full load condition

$$s = s_f, \quad T = T_{fl} \quad \text{and} \quad I_1 = I_{fl}$$

(1) $\Rightarrow$ becomes

$$\therefore T_{fl} = \frac{(I_{fl})^2}{s_f} \quad \rightarrow (3)$$

∴ Dividing (2) by (3)

$$\frac{T_{st}}{T_{fl}} = \frac{(n I_{sc})^2}{\frac{(I_{fl})^2}{s_f}}$$

$$\frac{T_{st}}{T_{fl}} = \left(\frac{n I_{sc}}{I_{fl}} \right)^2 \times s_f$$

$$\boxed{\frac{T_{st}}{T_f} = n^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 s_f}$$

Not:-

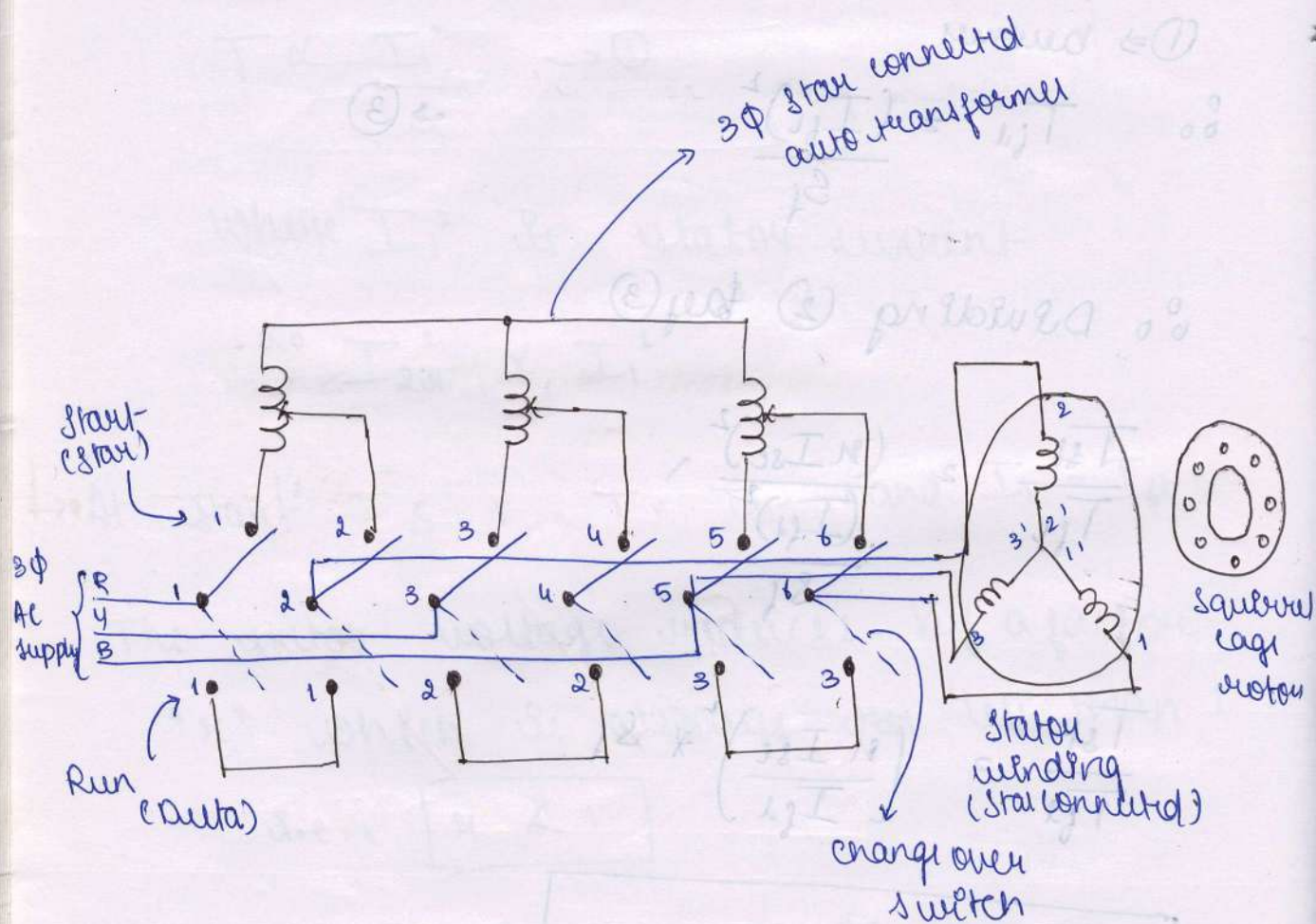
As $n < 1$, the starting reduces by a fraction n^2 due to stator

resistance ~~at~~ starter.

$$(T_{st} \propto n^2 T_f)$$

L2) Auto transformer starter :-

A three phase star connected auto-transformer is used to reduce the voltage applied to the stator



At start - switch $\Rightarrow$ Auto star ~~had~~ side
 (0-400) V can be varied
 ∴ current is reduced
 reduced voltage is applied by
 varying auto k from 0 to 400 V
 ∴ (change over switch in start condition)

- It consists of a suitable change
over switch

- when the switch is in start position, the winding is supplied with reduced voltage.

- This can be controlled by tapping provided in the autotransformer

- when motor gathers 80% of normal speed, change over switch is thrown into run position and the rated voltage gets applied to stator winding

→ Relationship b/w T_{st} and T_{FL} %

let 'x' be the fractional ~~per~~ percentage tapping used for the autotransformer

If I_{sc} = starting motor current at rated voltage

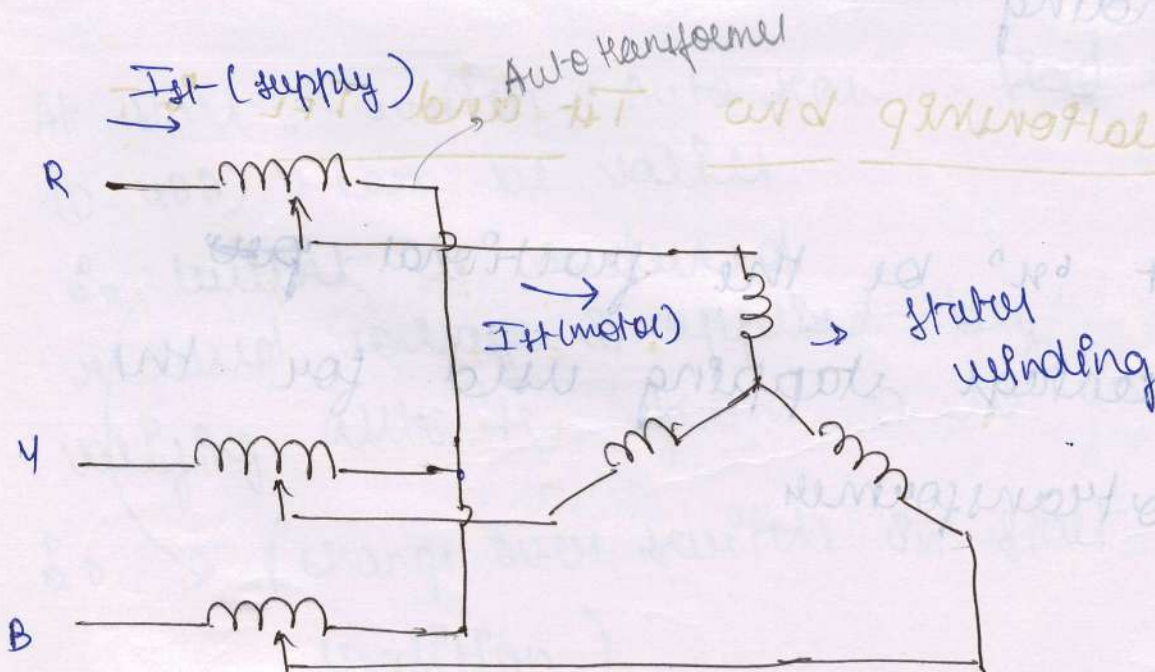
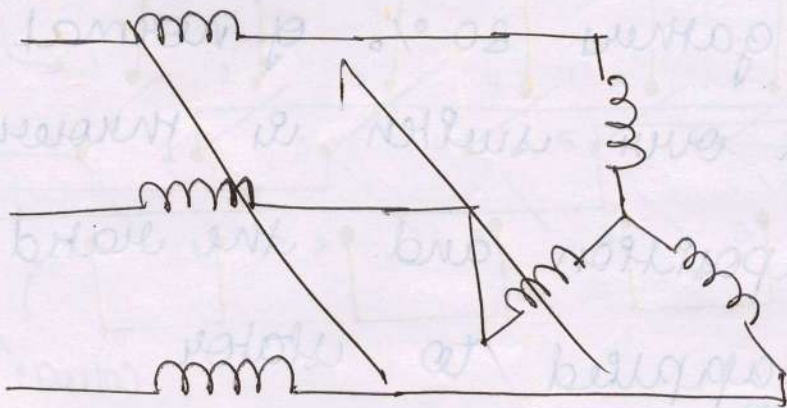
I_{st} = starting motor current with the starter

then

$$I_{st} = n I_{sc}$$

→ ①

n = fraction



There exists a fixed ratio ~~ratio~~ ratio between I_{st} (supply) and I_{st} (motor) due to auto transformer

$$\boxed{I_{st} \text{ (supply)} = n I_{st} \text{ (motor)}} \quad \text{--- (2)}$$

→ At start $s = 1$, $T = T_{st}$, $N = 0$

$$T_{st} \propto I_{st}^2 \text{ (motor)}$$

$$T_{st} \propto n^2 I_{sc}^2 \quad \rightarrow \text{--- (3)}$$

$$T_{fl} \propto \frac{I_{fl}^2}{s_f} \quad \rightarrow \text{--- (4)}$$

$$\frac{T_{st}}{T_{fl}} = \frac{n^2 I_{sc}^2}{\frac{I_{fl}^2}{s_f}}$$

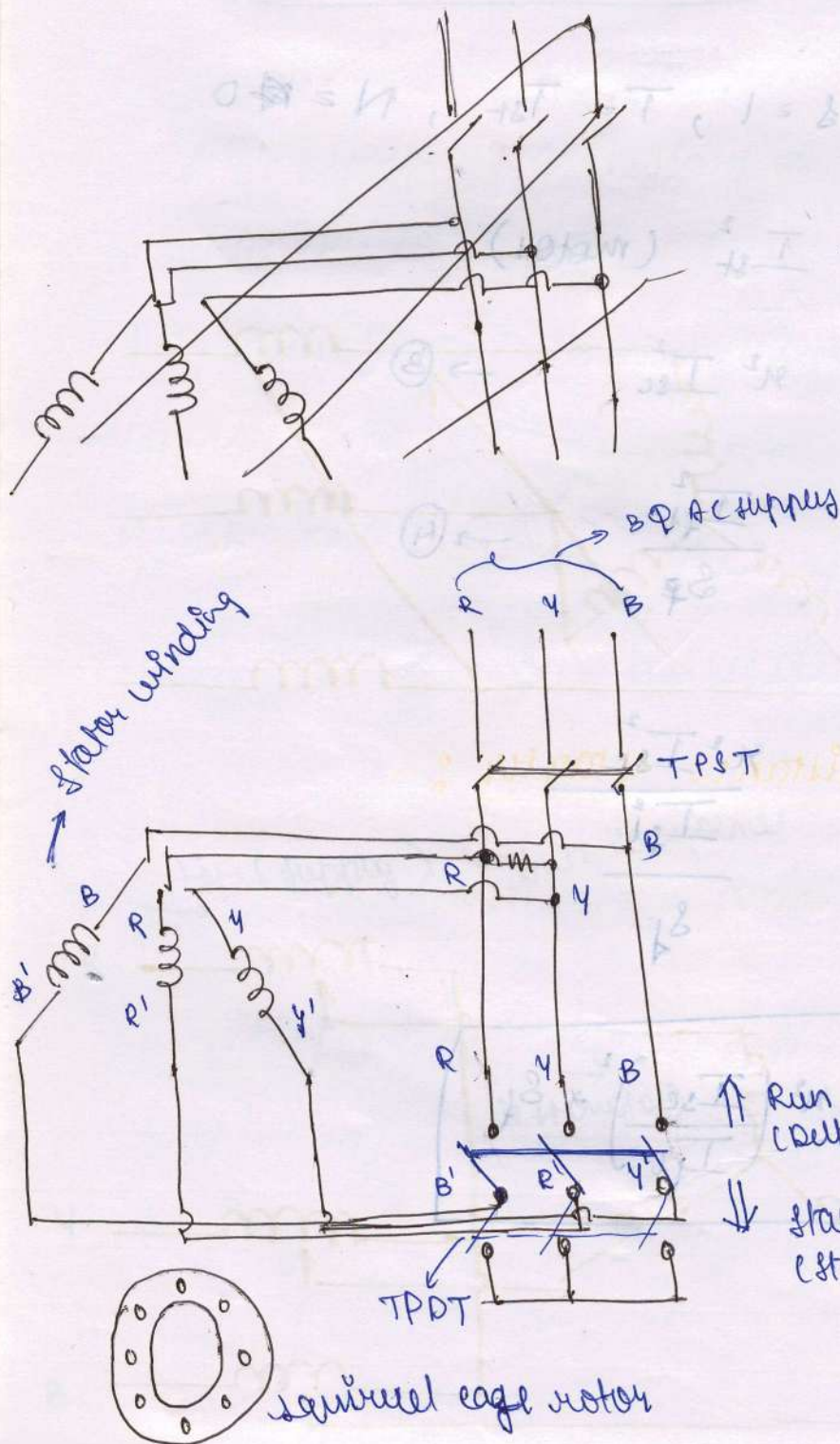
$$\boxed{\frac{T_{st}}{T_{fl}} = n^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 \times s_f}$$

Note

Starting torque reduced by $\frac{1}{3}$

3) Star - delta starter :-

Star	Delta
$V_{ph} = \frac{V_L}{\sqrt{3}}$	$I_{ph} = \frac{I_L}{\sqrt{3}}$
$I_{ph} = I_{Ln}$	$V_L = V_{ph}$



It gets normal voltage connected with motor at full speed

Due to this at start per phase voltage is limited hence starting current is reduced

V_{sc}

- Ratio of T_{st} and T_{fl}

→ Relationship between T_{st} and T_{fl} :-

W. K T on case of autotransformer
starter

$$\frac{T_{st}}{T_{fl}} = \kappa^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 S_f$$

Now the factor ' κ ' for star-delta
starter is $\frac{1}{\sqrt{3}}$

$$\kappa = 1/3$$

$$\therefore \frac{T_{st}}{T_{fl}} = \left(\frac{1}{3} \right) \left(\frac{I_{sc}}{I_{fl}} \right)^2 S_f$$

→ Rotor resistance starter :-

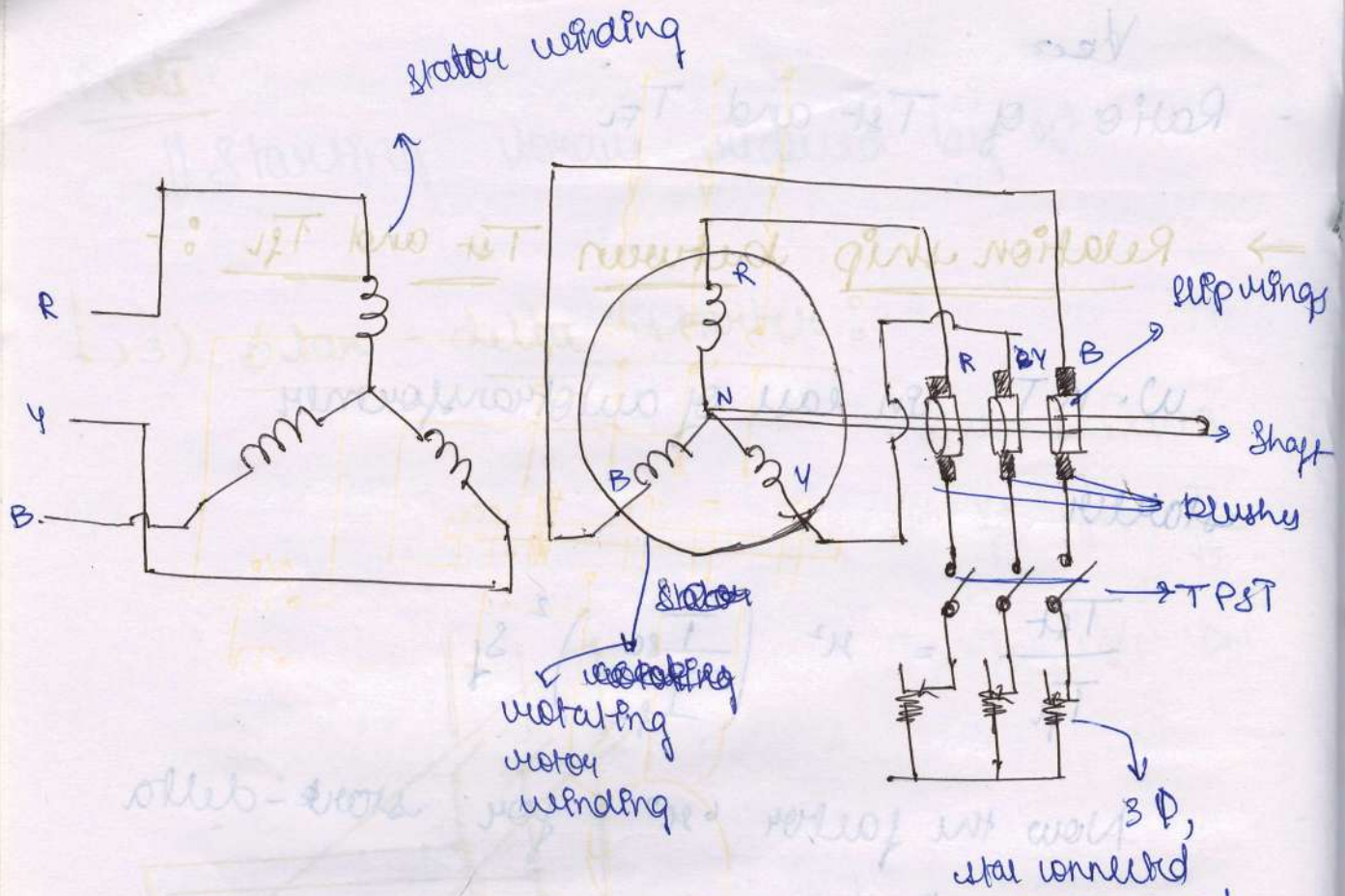
- This type of starter is applicable
only for slip ring induction motor.

Initially max resistance as motor gathers
speed resistance is removed

$$T \propto \frac{I_{sc}^2 R}{s} \rightarrow T_{st} \propto I_{sc}^2 R$$

as $R \uparrow \rightarrow T_{st}$ increases

Not applicable for squirrel cage motor



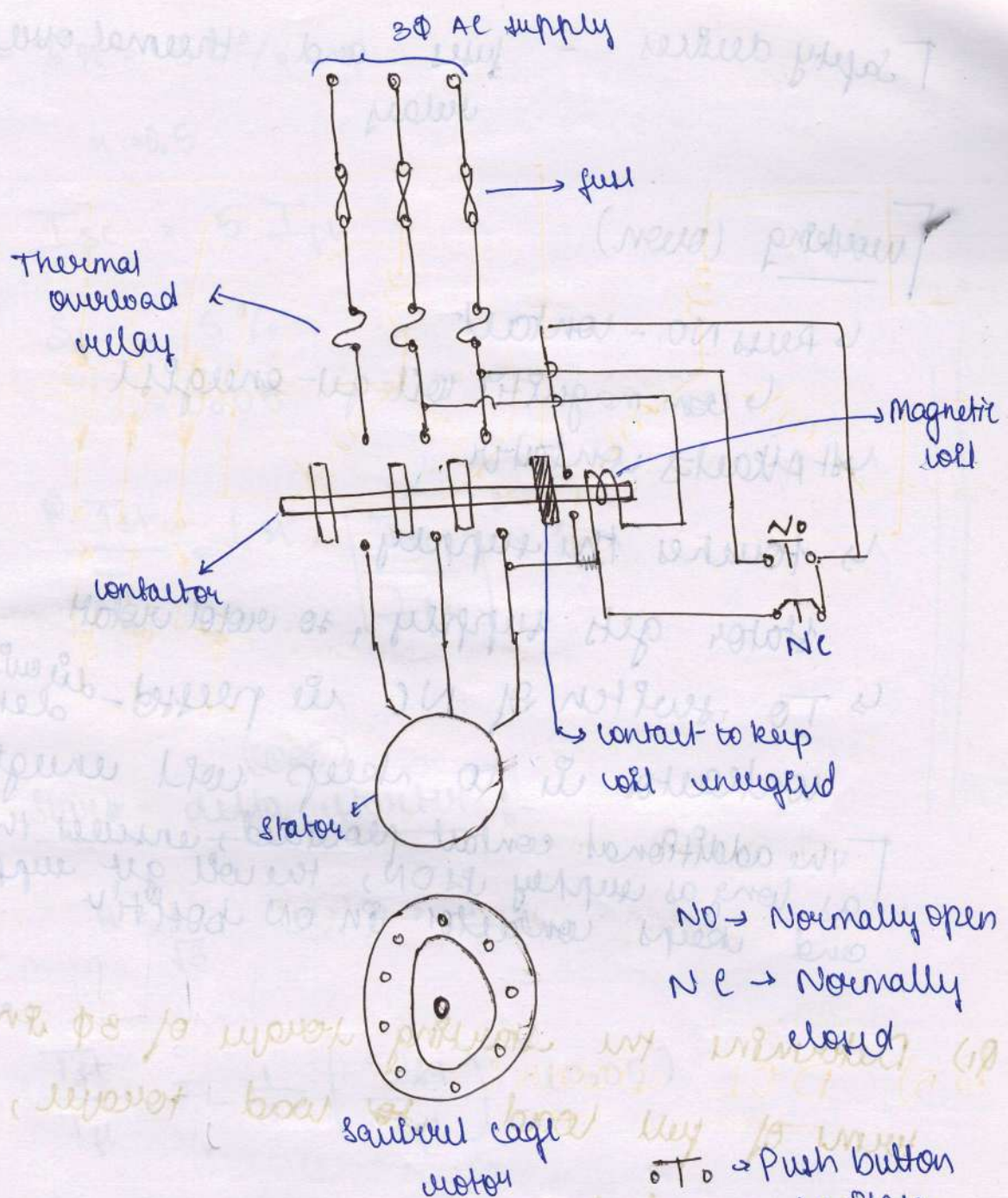
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At start station we should be stationary resistance load
 man take on internal resistance & reduced

↳ 5) DOL starter :-

It does not reduce the supply voltage.

[Direct on line starter]



Thermal relay - sensor → senses any abnormality]
 - Protective device

Thermal overload relay - senses change in high current
 w/ing temperature variation

// → Motor below 5HP - DOL starter

i.e., small rating motor - high starting is
 present in motor for only less duration of
 time ∴ does not affect the motor

→ Contactor - switch gear arrangement

2 Push button switches - NO & NC

Safety devices - fuses and thermal overload relay

Working (over)

↳ Press NO - contact

↳ ~~can~~ magnet will get energised

↳ attracts contactor

↳ touches the supply

stator gets supply, so motor rotates

↳ To switch off NC is pressed - circuit de-energised
contacts in to keep will energised

The additional contact provided, ensures that as long as supply is ON, the will get supply and keeps contactor in ON position

Q1) Determine the starting torque of 3 ϕ in terms of full load ~~to~~ load torque, when started by

(i) star delta starter

(ii) auto transformer with 50% tapping.

- The short circuit current of the motor is 5 times the full load current and full load slip is 5%

$$\underline{\text{Soln}} - \% n = 50\%$$

$$n = 0.5$$

$$I_{sc} = 5 I_{fl}$$

$$- S_f = 5\% \\ = 0.05$$

$$\rightarrow \textcircled{P} \cdot \frac{T_{st}}{T_{fl}} = (n^2) \left[\frac{I_{sc}}{I_{fl}} \right]^2 \times S_f$$

$\hookrightarrow \textcircled{i}$ Star-delta starter :-

$$n = \frac{1}{\sqrt{3}}$$

$$\text{or } n = \frac{1}{3}$$

$$\frac{T_{st}}{T_{fl}} = \frac{1}{3} \left(\frac{5 I_{fl}}{I_{fl}} \right)^2 (0.05)$$

$$T_{st} = 0.416 T_{fl}$$

$$T_{st} = 41.66 \% T_{fl}$$

$\hookrightarrow \textcircled{ii}$ Auto transformer starter

$$n = 0.5$$

$$\frac{T_{st}}{T_{fl}} = (0.5)^2 \left(\frac{5 I_{fl}}{I_{fl}} \right)^2 (0.05)$$

$$T_{st} = 0.3125 T_{fl}$$

$$\underline{T_{st} = 31.25\% T_{fl}}$$

Q2) A three phase 6 pole, 50 Hz induction motor takes 60 Amps at full load speed of 940 rpm and develops a torque of 150 Nm. The starting current at rated voltage is 300 Amps. What is the starting torque? If a star delta starter is used, determine the starting torque and starting current.

Soln:- // a) In case (i) starter is mentioned n^2 is not considered

Given:-

$$P = 6$$

$$f = 50 \text{ Hz}$$

$$I_{fl} = 60 \text{ A}$$

$$N_s = 940 \text{ rpm} \quad [(\omega) \text{ N}]$$

$$T_{fl} = 150 \text{ Nm}$$

$$I_{sc} = 300 \text{ A}$$

- To find :-
- (i) T_{st} = without any starter
 - (ii) T_{fl} = with star delta starter
 - (iii) I_{sc} = starting current with starter

Soln :-

(i) without starter

$$\frac{T_{st}}{T_{fl}} = \frac{1}{n^2} \left(\frac{I_{sc}}{I_{fl}} \right)^2 S_f \quad \text{(iii) } \leftarrow$$

$$N_{fl} = N_s (1 - S_f)$$

$$\rightarrow N_s = \frac{120 f}{P}$$

$$= \frac{120 \times 50}{6}$$

$$N_s = 1000 \text{ rpm}$$

$$\rightarrow 940 = 1000 (1 - S_f)$$

$$S_f = 0.06$$

$$\rightarrow \frac{T_{st}}{150} = \left(\frac{300}{60} \right)^2 \times 0.06$$

$$T_{st} = \underline{225 \text{ Nm}}$$

(ii) with star delta starter

$$n^2 = \frac{1}{3}$$

$$\rightarrow \frac{T_{st}}{150} = \frac{1}{3} \left(\frac{300}{60} \right)^2 0.06$$

$$\rightarrow T_{st} = 15 \text{ N-m}$$

→ (iii) starting current with started

$$\frac{T_{st}}{T_{fl}} = n^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

$$\sqrt{\frac{T_{st}}{T_{fl}}} = n \left(\frac{I_{sc}}{I_{fl}} \right) \sqrt{s_f}$$

$$I_{sc} = \frac{\sqrt{\frac{T_{st}}{T_{fl}}} \times I_{fl}}{n \sqrt{s_f}}$$

$$= \frac{\sqrt{\frac{15}{150}} \times 60}{\frac{1}{3} \times \sqrt{0.06}}$$

$$= \underline{\underline{300 \text{ A}}}$$

⇒ Speed control of three phase induction motor:-

WKT

$$N = N_s (1 - s)$$

(or)

$$s = \frac{N_s - N}{N_s}$$

From the above expression, it is seen that speed of the induction motor can be changed either by changing the synchronous speed N_s or by changing the slip (s).

Similarly, torque produced is given by -

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

∴ Parameters

E_2 and R_2 are changed, then to keep the torque constant in constant load condition, the

motor reacts by changing its
slip and in turn the speed changes
[N_s and slip]

→ Types of speed

Speed control of 3ϕ IM can be achieved
from two methods

↳ 1) From stator side

↳ 2) From motor side

1) From the stator side :-

- ↳ (i) Supply frequency control ^{to control} ~~by varying~~ N_s
called $\left(\frac{V}{f}\right)$ method

- ↳ (ii) Supply voltage control

- ↳ (iii) Adding ~~resistor~~ rheostat in the
stator circuit

- ↳ (iv) Controlling the no. of stator
poles to control N_s

2) From the motor side :-

- ↳ (i) Adding external resistance
in the motor circuit

• $\rightarrow$ (ii) Cascade control

• $\rightarrow$ (iii) Injecting slip frequency voltage on the motor circuit.

$\Rightarrow$ 1) From the stator side:-

• $\rightarrow$ (i) Supply frequency control or
(V/f) method:-

W.K.T

$$N_s = \frac{120f}{P}$$

Thus by controlling the supply frequency 'f' smoothly synchronous speed can be controlled over a wide range.

- The expression for air gap flux is given by -

$\rightarrow$ supply frequency in unit of V/Hz

$$\Phi_g = \frac{1}{k_m k_t T_p n} \left(\frac{V}{f} \right)$$

Due to transformer action

[For motor or generator = T = No. of turns
For transformer = N = No. of turns

$$\Phi_g = \frac{1}{k_m k_t T_p n} \left(\frac{V}{f} \right)$$

where

K_1 = Stator winding constant

T_{pn} = Stator turns per phase

V = Supply voltage

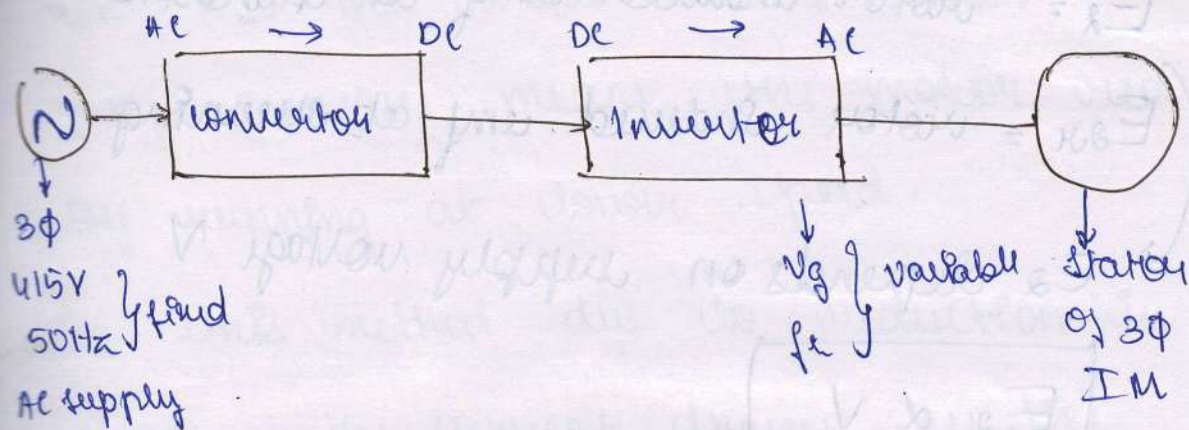
f = Supply frequency

- It can be seen that if the supply frequency f is changed, the value of air gap flux also gets affected.

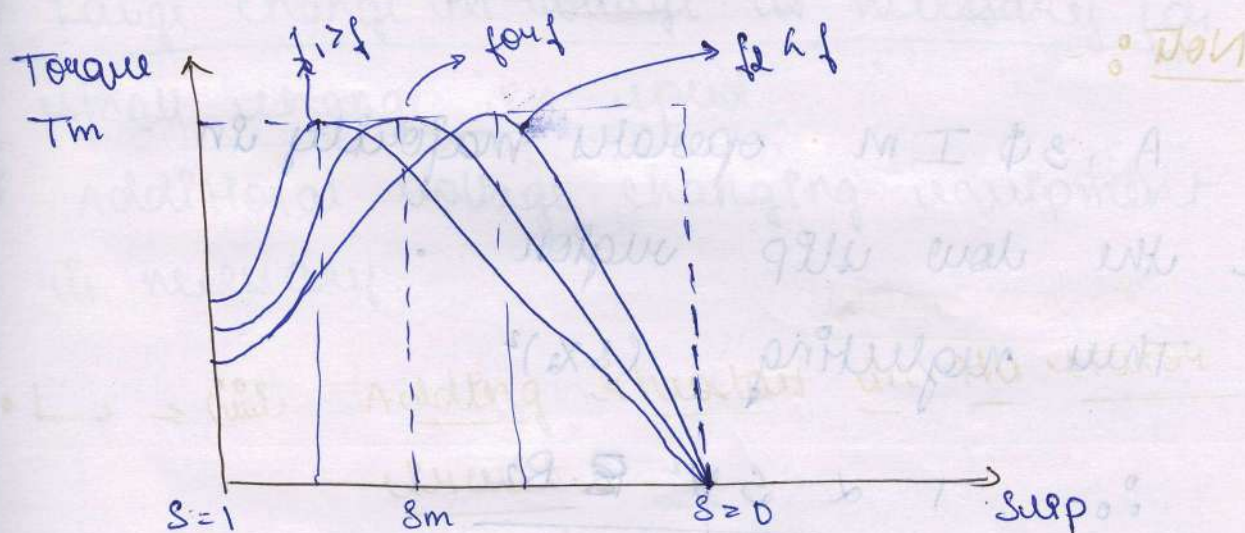
- This results ~~in~~ saturation of stator and motor ~~poles~~ cores and hence it is necessary to maintain air gap flux constant.

- To achieve this, along with frequency f , ϕ supply voltage V also must be changed and ratio of V/f should be maintained constant.

- Hence this method is called V/f control.



→ Torque slip characteristics:-



f = supply frequency

$f_1 = f_2$ = variable frequency

f_1 = frequency greater than supply frequency

f_2 = frequency less than supply frequency

• → Supply voltage control:-

$\omega \propto T$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

E_2 = motor induced emf at standstill

E_{2x} = motor induced emf at running

E_2 depends on supply voltage V

$$E_2 \propto V$$

— For low slip region $(sX_2) \ll R_2$

Note :-

A 3 ϕ IM operates majority in the low slip region.

Thus neglecting $(sX_2)^2$

$$\therefore T \propto \frac{s V^2 R_2}{R_2^2}$$

assume R_2 ~~remains~~ remains constant (squirrel cage motor)

$$\therefore T \propto s V^2$$

— If the supply voltage is V in reduced ~~the~~, torque also decreases.

* — But to sustain the same load it is necessary to develop the same torque, hence slip increases

is that the torque remains same.

Slip increases means the motor runs by running at lower speed.

→ In this method due to reduction in voltage, the current drawn by the motor increases

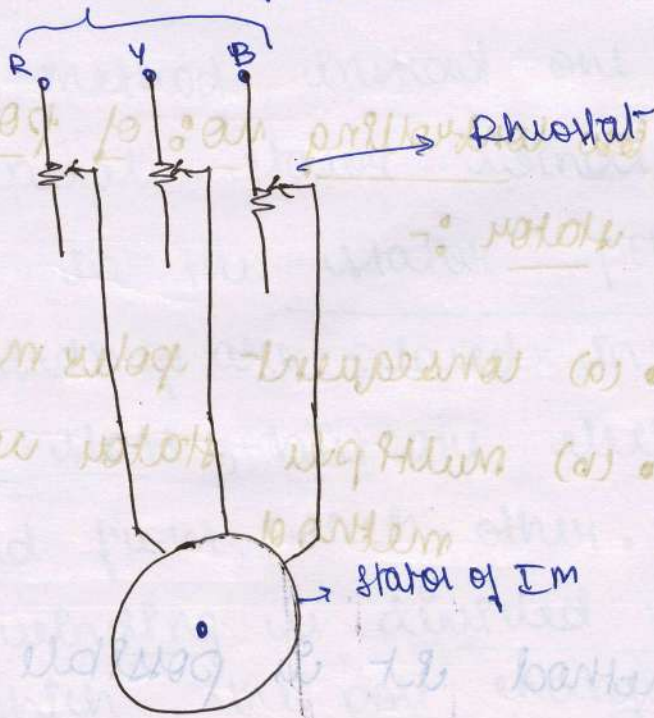
* Large change in voltage is necessary for small change in speed

* Additional voltage changing equipment is necessary.

• 1 → (iii) Adding rheostat in the stator

Result :-

3 ϕ supply



squirrel cage motor

- Reduced voltage can be applied to the stator by adding rheostat in the stator circuit.
- A part of the supply voltage gets dropped across the rheostat and in turn reduces the voltage applied to the stator.
- This causes a reduction in speed.

Disadvantage

- There are large power loss due to the rheostats.

• $\rightarrow \rightarrow$ (ii) controlling no. of poles in the stator :-

- $\rightarrow \rightarrow \rightarrow$ • (a) consequent poles method
- $\rightarrow \rightarrow \rightarrow$ • (b) multiple stator winding method

- In this method it is possible to have 1, 2, or 4 speeds in steps, by changing the no. of stator poles.

- A continuous smooth speed control is not possible by this method.

↳ → (a) consequent poles method:-

- In this method, connections of the stator winding are changed with the help of simple switching technique.
- Due to this, the no% of stator poles gets changed in the ratio 2:1.
- Hence, either of the two synchronous speeds can be selected.

↳ → (c) multiple stator winding method:-

- In this method instead one winding, two separate stator windings are placed in the stator poles.
- The windings are placed in the same stator slots, but are electrically isolated from each other.
- Each winding is divided into cold to which the pole changing with consequent pole method is provided.

Limitations

- 1) can be applied to squirrel cage motors only
- 2) Smooth speed control is not possible, only steps changes in speed is possible
- 3) Two different stator windings are required to be wound, which increases the cost.
- 4) complicated design

Speed control from motor side :-

Adding internal resistance to motor circuit -

WKT

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

for low slip region

$$(s X_2) \ll R_2$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2}$$

if E_2 is assumed to be constant for constant supply V_g

$$T \propto \frac{s}{R_2}$$

So As $R_2 \downarrow \rightarrow T \uparrow$

as $s \downarrow \rightarrow \text{speed} \downarrow$

smooth speed control below rated speed is possible.

$R_2 \uparrow$
 $T \downarrow$ but $s \uparrow$
 so speed $\uparrow$

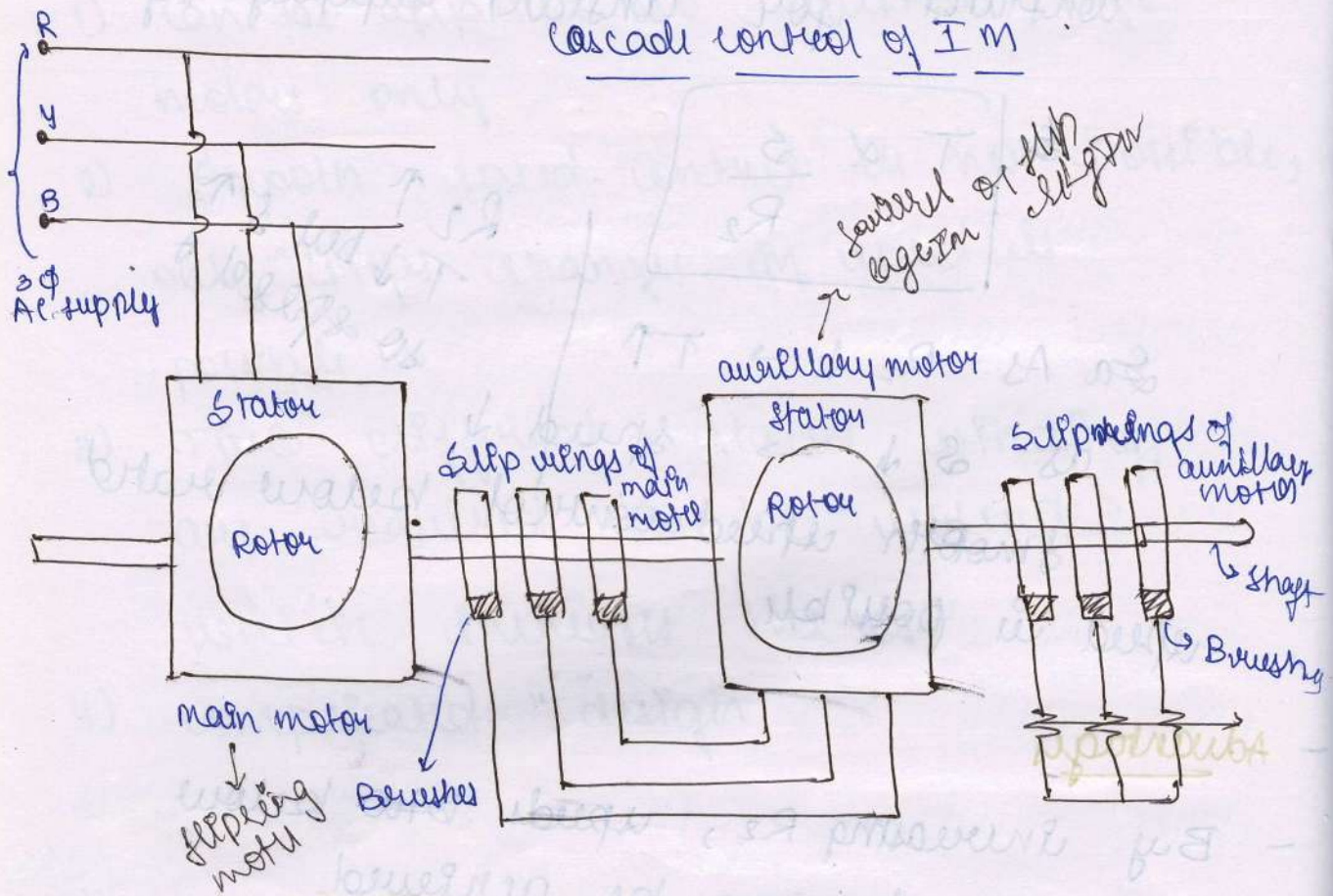
- Advantages

- By increasing R_2 , speeds ~~to~~ below normal speed can be achieved
- The starting torque of the motor increases proportional to the increase in motor resistance.

- Disadvantages

- Large speed changes are not possible
- cannot be used for squirrel cage motor IM
- Speed above normal speed cannot be achieved
- Large power loss due to I^2R losses
- Efficiency is low.

$\rightarrow \rightarrow \rightarrow \omega$ Cascade control :-



- In this method, 2 IM are mounted on the same shaft, first motor called the main motor is a slipping IM.
- Second motor called Auxiliary motor can be slip ring or squirrel cage IM.

operation

- The supply to the main motor is from 3φ AC.
- The supply to the auxiliary motor is derived at a slip frequency from the slip rings of main motor.

- This is called cascading of motors
- If the torque produced by the motors act in the same direction, then the motor resistance decreases and is called additive cascading

- If the torque produced by the motors are in opposite direction then the motor resistance increases called differential cascading

Let

P_A = no. of poles of the main motor

P_B = no. of poles of the auxiliary motor

f = frequency of supply

f_A = frequency of ~~rotor~~ motor

induced emf of main motor

f_B , [frequency of rotor induced emf of auxiliary motor]

// i.e. input frequency to the auxiliary motor

∴ $f_A = f_B$

N = speed of the ~~rotor~~ motor set

$$\frac{1000}{60} \times \frac{60}{100} = 100$$

→ Synchronous speed of main motor

$$N_{SA} = \frac{120 f}{P_A}$$

$$N_{SA} = \frac{120 f}{P_A} \quad \text{--- (m)}$$

slip of main motor

$$\% S_A = \frac{N_{SA} - N}{N_{SA}} \times 100$$

$$S_A = \frac{N_{SA} - N}{N_{SA}} \quad \rightarrow \textcircled{1}$$

$$f_A = S_A f$$

W.K.T

$$f_B = f_A = S_A f$$

→ Synchronous speed of auxiliary motor :-

$$N_{SB} = \frac{120 f_{SA}}{P_B}$$

$$= \frac{120 f_B}{P_B}$$

$$N_{SB} = \frac{120 S_A f}{P_B} \quad \rightarrow \textcircled{2}$$

$$N_{SA} = \frac{120 f}{P_A}$$

$$S_A = \frac{N_{SA} - N}{N_{SA}}$$

$$f_A = S_A f = S_A f$$

$$N_{SB} = \frac{120 f_A}{P_B}$$

$$= \frac{120 S_A f}{P_B}$$

$$= \frac{120 f}{P_B} \left(\frac{N_{SA} - N}{N_{SA}} \right)$$

Sub ① un(2)

$$N_{SB} = \frac{120}{P_B} \left[\frac{N_{SA} - N}{N_{SA}} \right]$$

under no load condition

$$N \approx N_{SB}$$

$$N = \frac{120}{P_B} \left[\frac{N_{SA} - N}{N_{SA}} \right]$$

$$= \frac{120}{P_B} \left[1 - \frac{N}{N_{SA}} \right]$$

∴ from (m)

$$N = \frac{120}{P_B} \left[1 - \frac{N}{\frac{120}{P_A}} \right]$$

simplifying

$$N = \frac{120}{P_A + P_B}$$

for cumulative cascading

$$N = \frac{120}{P_A - P_B}$$

for differential cascading

Thus the speed of ω_n is decided
by total no. of poles equal to
 $P_A + P_B$ for ~~some~~ cumulative cascading
 $P_A - P_B$ for differential cascading

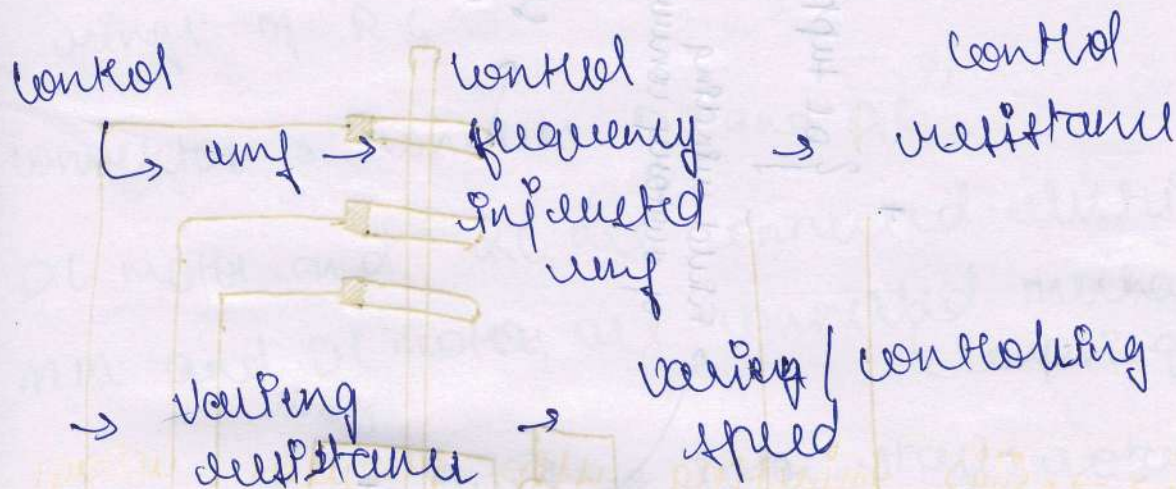
Method (iii) Injecting slip frequency voltage
in the motor circuit :-

In this method voltage / emf having
the slip frequency is injected
into the motor circuit.

- It is possible that the injected
slip frequency voltage may oppose
the motor induced emf then the
effective motor resistance increases

- If the injected slip frequency
voltage assists or helps the
motor induced emf (in phase) then
effective motor resistance decreases

$\text{emf} \rightarrow \begin{cases} \text{Rotor induced emf} \\ \text{external injected emf} \end{cases}$



There are two methods available for this principle -

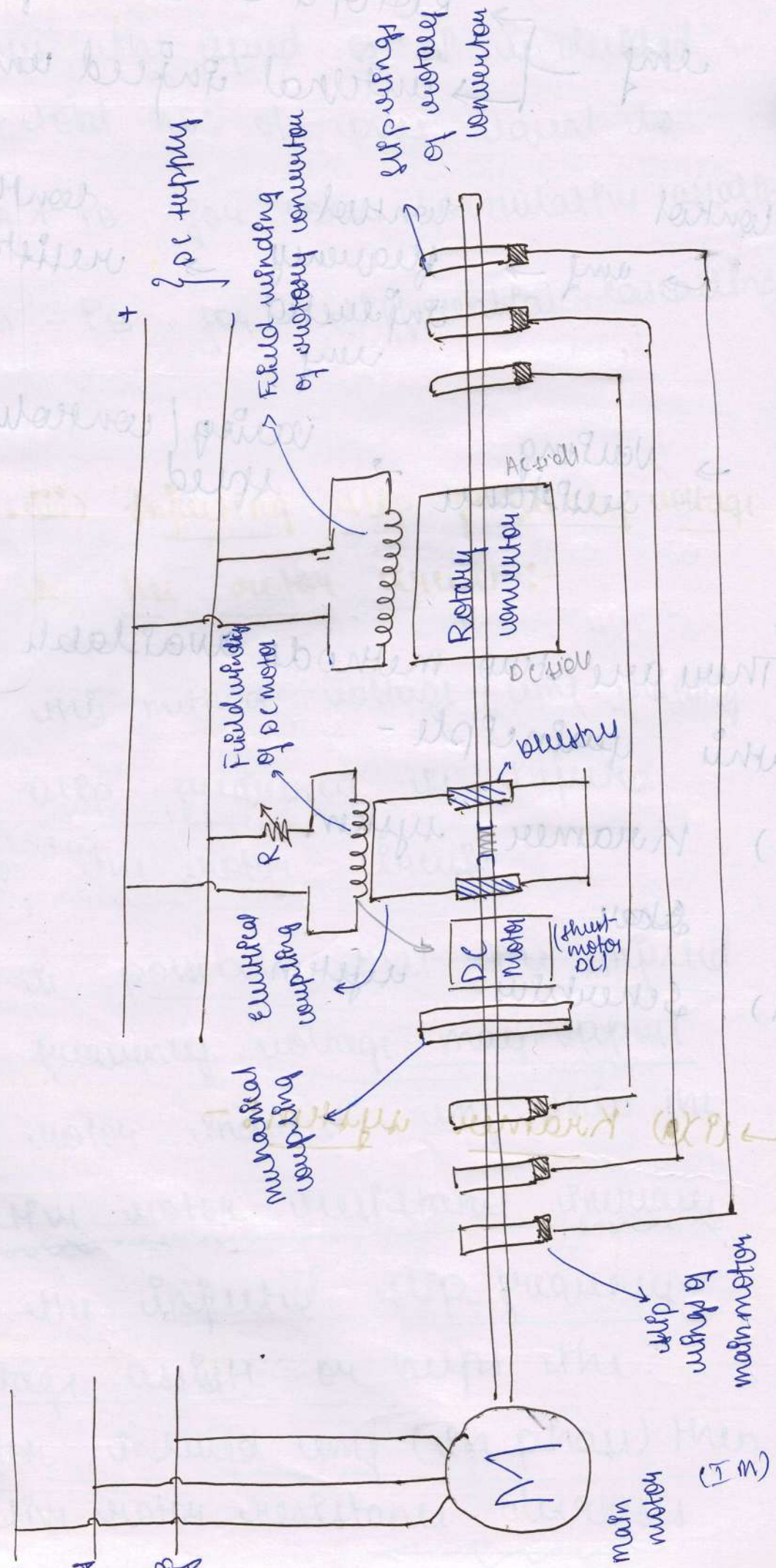
- (i) Kramer system
- (ii) Scherbius system

Inject. of su
ven shou

→ (ix) Kramer system:-



Kramer system



slip frequency ^{emb} is needed to be injected

slip rings of MM is connected to slip rings of RC

converter $\rightarrow$ converts DC and AC

DC motor and RC are connected electrically
MM and DC motor are connected mechanically (coupling)

$\rightarrow$ RMP is produced $\rightarrow$ motor starts rotating
2 additional motor / equipment $\rightarrow$ work
in motor

$\rightarrow$ vary $\rightarrow$ R $\rightarrow$ ^{short circuit} current changes (AC)
flux change $\rightarrow$ ^{induced} EMF is produced
then given to RC through RC brushes
RC convert ~~AC~~ DC to AC

(speed remains same throughout as
in main motor
 $\rightarrow$ From RC $\rightarrow$ to slip ring goes
to main motor so injecting

frequently into the main motor
then $\rightarrow$ motor induced EMF $\rightarrow$ if these two
 $\rightarrow$ slip ring frequency changes slip
changes,

- Advantages
- Smooth speed control is possible
- wide range of speed control is possible

- Design of motor converter is independent of speed control.

- Disadvantage

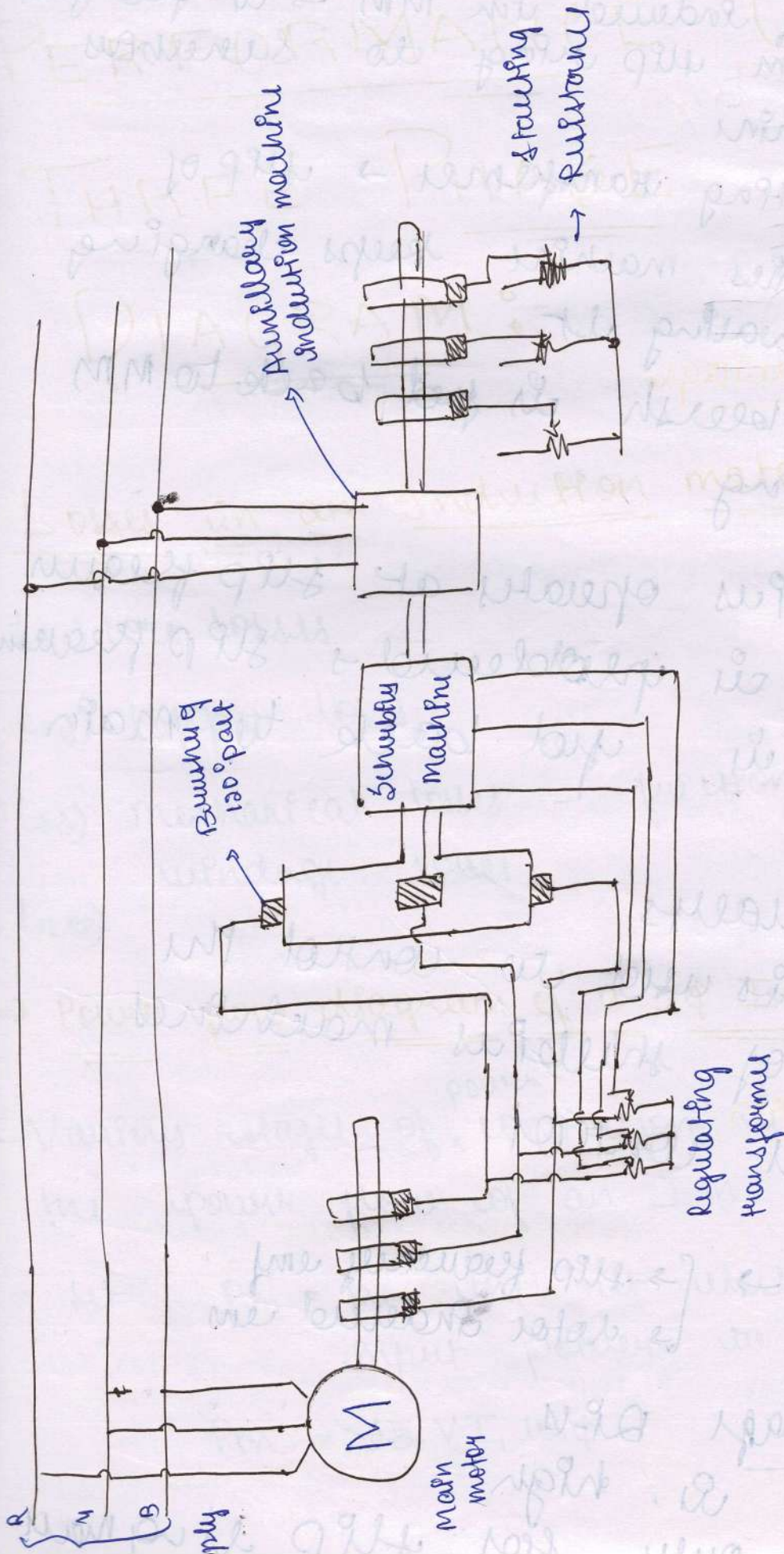
1) - It can be used only ~~slip~~ for speed control slip using IM

- To control the speed of motor two additional equipments i.e., a DC shunt motor and a motor converter is required which increases the overall cost.

- Application

- used for speed control of motor of 4000 kW in steel rolling mill

↳ (iii) • Scherbius system :-



→ 3 ϕ AC supply → main motor → speed to be controlled
 → From slip ring to Scherbius machine

Regulating transformer → slip of Scherbius machine keeps changing by varying it
 from brush is fed back to MM slip ring

Scherbius operates at slip frequency
 → emf in ~~problem~~ slip frequency
 emf is fed back to main motor

→ Auxiliary ω is used to control the slip of Scherbius machines under control

at MM → $\left\{ \begin{array}{l} \rightarrow \text{slip frequency emf} \\ \rightarrow \text{rotor induced emf} \end{array} \right.$

Disadvantage DPs high
 cost ω high
 slip ring machine

⇒ MODULE - 3

PERFORMANCE OF

THREE PHASE

DIAGRAM :-

Losses in an Induction motor :-

↳ 1) core losses

↳ 2) copper losses

↳ 3) mechanical losses - friction and
windage losses

↳ 4) ~~losses~~

⇒ Power flow diagram of a 3 ϕ IM :-

- Various stages of ^{power} conversion is called the power flow of an IM.

- Let P_{in} = be the net electrical input power to the motor.

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

where $V_L =$ line voltage

$I_L =$ line current

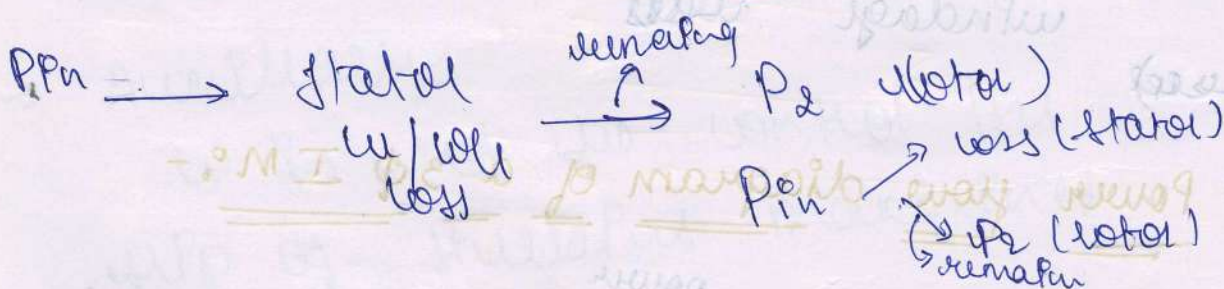
$\cos \phi =$ Power factor of the motor

This is nothing but stator input

- Part of this i/p power is utilised to supply the stator core loss as well as copper loss
- The remaining power is delivered to the motor magnetically through the air gap with the help of

RMF

- This is called motor input power denoted by $= P_2$



$$P_2 = P_{in} - (\text{stator core} + \text{stator copper loss})$$

$$P_{in} = P_2 + \text{stator \& Rotor Cu losses}$$

- The motor is not able to convert its entire input (P_2) to mechanical power as has to supply motor losses

- The motor iron losses are very small because of motor frequency (f_a) is very less

$$f_a = \frac{s}{60} \rightarrow \text{very small}$$

(s is in fraction value)

- Thus effectively the motor P/P (P_2) has to supply mainly the motor copper losses only

- Thus motor copper losses are given by

$$P_{cu} = 3 I_{2a}^2 R_2 \rightarrow \text{(for slip ring only)}$$

↳ due to R_2)

I_{2a} = motor current under running

R_2 = Rotor resistance

$$P_m = P_2 + \text{stator loss}$$

$$P_2 = 3 I_{2a}^2 R_2$$

$$P_2 = P_m + P_{cu}$$

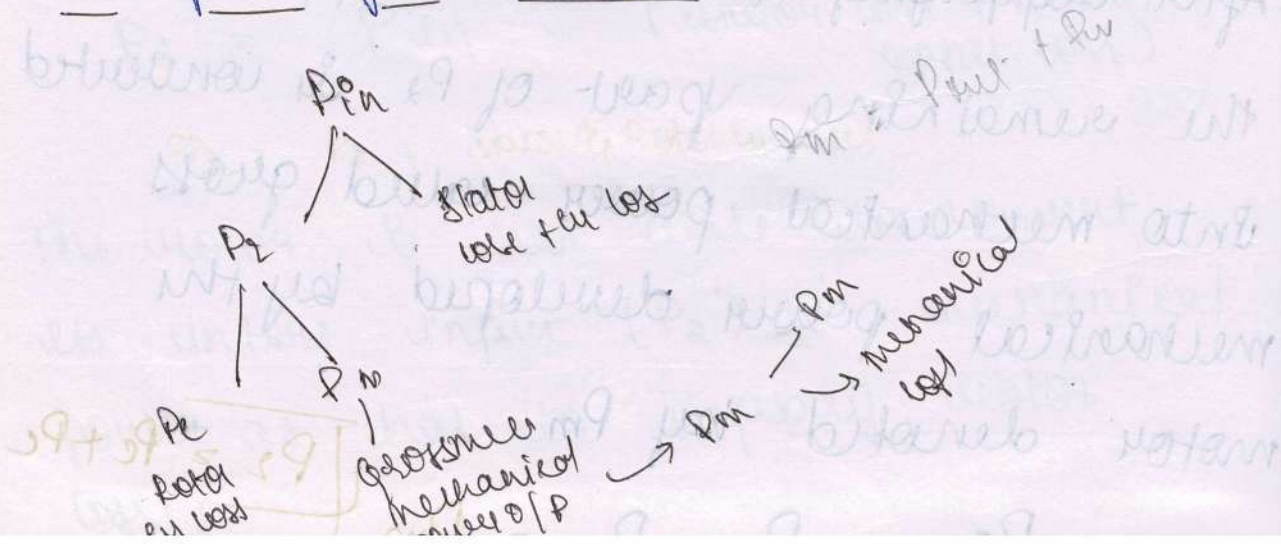
After supplying the motor copper loss the remaining part of P_2 is converted into mechanical power called gross mechanical power developed by the motor denoted by P_m

$$P_2 = P_c + P_{cu}$$

- This power P_m , the motor tries to deliver to the load connected via the shaft.
- But during this mechanical transmission, part of P_m is utilised to overcome the mechanical losses (Friction + windage).
- Finally the power available to the load at the ~~shaft~~ end of the shaft is called net output power of the motor denoted by ' P_{out} '

$$P_{out} = P_m - \text{mechanical losses}$$

* The power flow diagram is as shown -



Now the motor input - P_2 is

P_2 through RMF which is rotating at synchronous speed N_s .

Thus the corresponding torque developed by the motor and P_2 is given by

$$P_2 = T \times \omega_s$$

where $\omega_s = \frac{2\pi N_s}{60} \text{ rad/sec}$

$N_s = \text{rpm}$

$$P_2 = T \times \frac{2\pi N_s}{60} \quad \text{--- (1)}$$

The motor tries to deliver this torque to the load

Thus the motor output P_m and torque T is related by

$$P_m = T \times \omega$$

$$P_m = T \times \frac{2\pi N}{60} \quad \text{--- (2)}$$

New

~~P_m~~

$$P_m = P_2 - P_c$$

$$P_c = P_2 - P_m$$

$$\Rightarrow \cancel{P_c} = \frac{T \times 2\pi N_s}{60} - \frac{T \times 2\pi N}{60}$$

$$\boxed{P_c = \frac{T \times 2\pi}{60} [N_s - N]} \rightarrow (3)$$

↪ motor in loss

Dividing eqn (3) by (1)

$$\frac{P_c}{P_2} = \frac{\frac{T \times 2\pi}{60} [N_s - N]}{\frac{T \times 2\pi}{60} \times N_s}$$

$$\frac{P_c}{P_2} = \frac{N_s - N}{N_s}$$

$$\frac{P_c}{P_2} = s$$

$$\boxed{P_c = s P_2}$$

$$P_2 = \frac{T \times 2\pi N_s}{60} \quad P_m = \frac{T \times 2\pi N}{60}$$

$$P_m = s N_s \omega = P_c$$

$$P_m = P_2 - P_c$$

$$= P_2 - s P_2$$

$$P_m = P_2 (1 - s)$$

$$\frac{P_m}{P_2} = (1 - s)$$

$$P_2 : P_c : P_m = 1 : s : (1 - s)$$

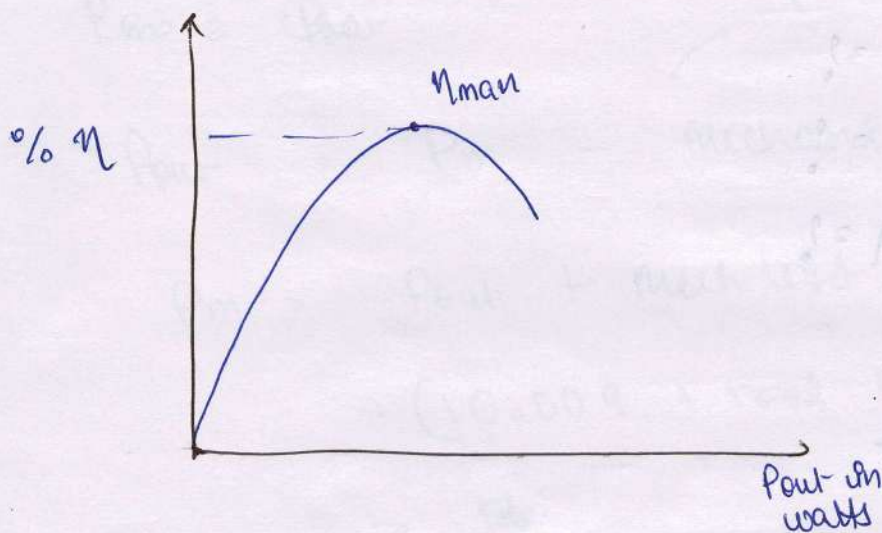
Note:-

The load torque is the net output torque or useful torque given by

$$T_{sh} = \frac{P_{out}}{\omega} \quad \text{N-m}$$

⇒ Efficiency of an induction motor:-

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100$$



Q1) A 4-pole, 50 Hz, 3 ϕ IM, running on full load with 4% slip develops a torque of 149.3 N-m at its pulley rim (CTR). The friction and windage losses are 200 W, the stator copper and iron ~~loss~~ losses are 1620 W, calculate.

- (i) D/P power
- (ii) Rotor cu losses
- (iii) Efficiency at full load

Given

$$P = 6$$

$$f = 50 \text{ Hz}$$

$$s = 4\% = 0.04$$

$$T_{\text{sh}} = \text{pulley rim torque} = 149.3 \text{ N-m}$$

$$P_{\text{fw}} = 200 \text{ W} = \text{friction and windage losses}$$

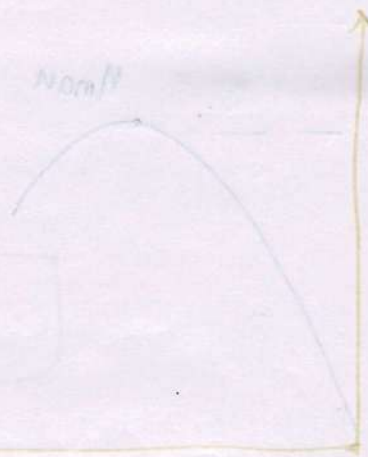
$$P_{\text{d}} = 1620 \text{ W} = \text{stator iron and cu losses}$$

To find :-

(i) $P_{out} = ?$

(ii) $P_e = ?$

(iii) $\% \eta = ?$



$$\rightarrow N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{2}$$

$$= 1000 \text{ rpm}$$

$$\rightarrow s = \frac{N_s - N}{N_s}$$

$$N = N_s - s N_s$$

$$= 1000 - 0.04 \times 1000$$

$$= 960 \text{ rpm}$$

$$\rightarrow P_{out} = T \times \omega$$

$$= 149.3 \times \frac{2\pi \times 960}{60}$$

$$= \underline{\underline{15.009 \text{ kW}}}$$

$$P_e = \frac{s}{1-s}$$

$$= 0.04$$

$$P_{me} = P_{out}$$

from power flow diagram

$$P_{out} = P_m - \text{mechanical loss}$$

$$P_m = P_{out} + \text{mech loss}$$

$$= (15.009 + 1.2 \times 0.2) \text{ kW}$$

$$= 15.209$$

$$P_m = \underline{\underline{15.209 \text{ kW}}}$$

$$\rightarrow \frac{P_e}{P_m} = \frac{s}{1-s}$$

$$P_e = \frac{15.209 \times 0.04}{1 - 0.04}$$

$$= 0.6337 \text{ kW}$$

$$P_e = \underline{\underline{633.7 \text{ Watts}}}$$

$$\rightarrow \% \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$P_2 = P_{in} - \text{stator iron / cu loss}$$

$$P_{in} = P_2 + \text{loss}$$

$$\frac{P_2}{P_e} = \frac{1}{s}$$

$$P_2 = \frac{P_e}{s}$$

$$= \frac{633.7}{0.04}$$

$$= 15.84 \text{ kW}$$

$$P_{in} = (15.84 + 1.62) \text{ kW}$$

$$= \underline{\underline{17.46 \text{ kW}}}$$

$$\% \eta = \frac{15.009}{17.46} \times 100$$

$$= \underline{\underline{85.96\%}}$$

Q2) A 6 pole Δ IM develops 30 HP including mechanical losses of 2 HP at a speed of 950 rpm on 550 V, 50 Hz supply. Power factor is 0.58. Calculate for this load

- (i) Slip
- (ii) Rotor copper loss
- (iii) Total i/p if stator losses are 2000 W
- (iv) The efficiency
- (v) The line current

Given

$$P = 6$$

$$P_{out} = 30 \text{ HP}$$

$$= 30 \times 735.5$$

$$= \underline{22065 \text{ watts}}$$

$$f = 50 \text{ Hz}$$

$$N = 950 \text{ rpm}$$

$$\text{mechanical losses} = 2 \text{ HP}$$

$$= 2 \times 735.5$$

$$= \underline{1471 \text{ watts}}$$

$$V_L = 550 \text{ V}$$

$$\cos \phi = 0.58$$

$$\text{stator loss} = 2 \text{ kW}$$

To find :-

$$(i) \quad s = ?$$

$$(ii) \quad P_c = ?$$

$$(iii) \quad P_{in} = ?$$

$$(iv) \quad \% \eta$$

$$(v) \quad I_L$$

$$(vi) \quad f_m$$

$$\begin{aligned} \rightarrow N_s &= \frac{120f}{P} \\ &= \frac{120 \times 50}{6} \\ &= 1000 \end{aligned}$$

$$\begin{aligned} \rightarrow & \cancel{8 \times 10^3 (1-s)} \\ &= 4000 (1-s) \end{aligned}$$

$$\rightarrow s = \frac{N_s - N}{N_s}$$

$$= \frac{1000 - 950}{1000}$$

$$s = \underline{0.005}$$

$$\rightarrow P_e$$

$$P_m = P_{out} + \text{Mech loss}$$

$$= 22065 + 1471$$

$$= \underline{230536 \text{ KW}}$$

$$\rightarrow \frac{P_e}{P_m} = \frac{s}{1-s}$$

$$P_e = \frac{(0.005)(230536)}{(1-0.005)}$$

$$= \underline{10238 \text{ KW}}$$

$$\rightarrow P_{in} = P_2 + \text{lw loss}$$

$$P_2 = \frac{P_e}{s}$$

$$= 24076$$

$$P_{in} = 24076 + 2$$

$$= \underline{26076 \text{ KW}}$$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{220065}{26076} \times 100$$

$$\% \eta = \underline{\underline{82045\%}}$$

$$\rightarrow P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$I_L = \frac{P_{in}}{\sqrt{3} V_L \cos \phi}$$

$$= \frac{267740.6}{\sqrt{3} \times 550 \times 0.58}$$

$$= \underline{\underline{48.45 \text{ A}}}$$

$$\rightarrow f_m = sf$$

$$= 0.005 \times 50$$

$$= \underline{\underline{205 \text{ Hz}}} = \underline{\underline{205 \text{ cycles/sec}}}$$

$$= 205 \times 60 \text{ cycles/min}$$

$$f_m = \underline{\underline{150 \text{ cycles/min}}}$$

Q3) A 25 kW, 4 pole, 3 phase 50 Hz IM is running at 1410 rpm supply full load. The mechanical losses are 850 W and stator losses are 1.07 times motor Cu loss on full load.

Calculate

- (i) Gross mechanical power developed
- (ii) motor copper losses
- (iii) Value of motor resistance / phase
if the motor current on full load / phase is 55 A
- (iv) Full load efficiency

Soln

Given

$$P_{\text{out}} = 25 \text{ kW} = P_{\text{out}}$$

$$P = 4$$

$$f = 50 \text{ Hz}$$

$$N = 1410 \text{ rpm}$$

$$\text{Mech loss} = 850 \text{ W}$$

$$\text{stator loss} = 1.07 \times \text{Cu loss}$$

To find

$$- P_m$$

$$- P_c$$

$$- R$$

$$N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{4}$$

$$= 1500 \text{ rpm}$$

$$s = \frac{1500 - 1410}{1500}$$

$$= 0.06$$

$$P_m = P_{out} + \text{mechanical loss}$$

$$= (25 + 111 \times 0.85) \text{ kW}$$

$$= 25$$

$$[A \cdot P_c \cdot P_m = 1.55\%]$$

$$\Rightarrow \frac{P_c}{P_m} = \frac{s}{1-s}$$

$$P_c = \frac{25.85 \times 0.06}{1 - 0.06}$$

$$= 1.65 \text{ kW}$$

$$\rightarrow P_c = 3 I_{2u}^2 R$$

$$\rightarrow R_2 = \frac{P_e}{3 I_{2s}^2}$$

$$= \frac{165000}{3 \times (65)^2}$$

$$= \underline{0.195 \text{ } \Omega} \quad \underline{0.130 \text{ } \Omega}$$

$$\rightarrow \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$P_{in} = P_2 + \text{cu loss}$$

$$P_2 = \frac{P_e}{s}$$

$$= \underline{2675}$$

$$= 2705 \text{ kW}$$

$$\text{stator loss} = 1.7 \times \text{cu loss}$$

$$= 1.7 \times P_e$$

$$= 1.7 \times 1650$$

$$= 2805$$

$$= 2.805 \text{ kW}$$

$$P_{in} = 2705 + 2.805$$

$$= 30.30$$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{25}{30.3} \times 100$$

$$= \underline{\underline{82.49\%}}$$

⇒ Equivalent circuit of a 3 ϕ IM:-

- 3 ϕ IM can be treated as a ~~generalized~~
generalized transformer

- If E_1 = emf induced in stator per phase

E_2 = Rotor induced emf ~~per~~ per phase
on standstill

Transformation ratio ~~$K = \frac{\text{stator turns}}{\text{rotor turns}}$~~

$$K = \frac{\text{rotor turns}}{\text{stator turns}}$$

$$K = \frac{E_2}{E_1}$$

If V_1 is the supply voltage per phase given to the stator, it produces the flux (RMF) which links with both stator and rotor.

In running condition E_2 becomes E_{2u} .

$$E_{2u} = s E_2$$

Now E_{2u} = motor induced emf per phase in running condition.

R_2 = motor resistance per phase

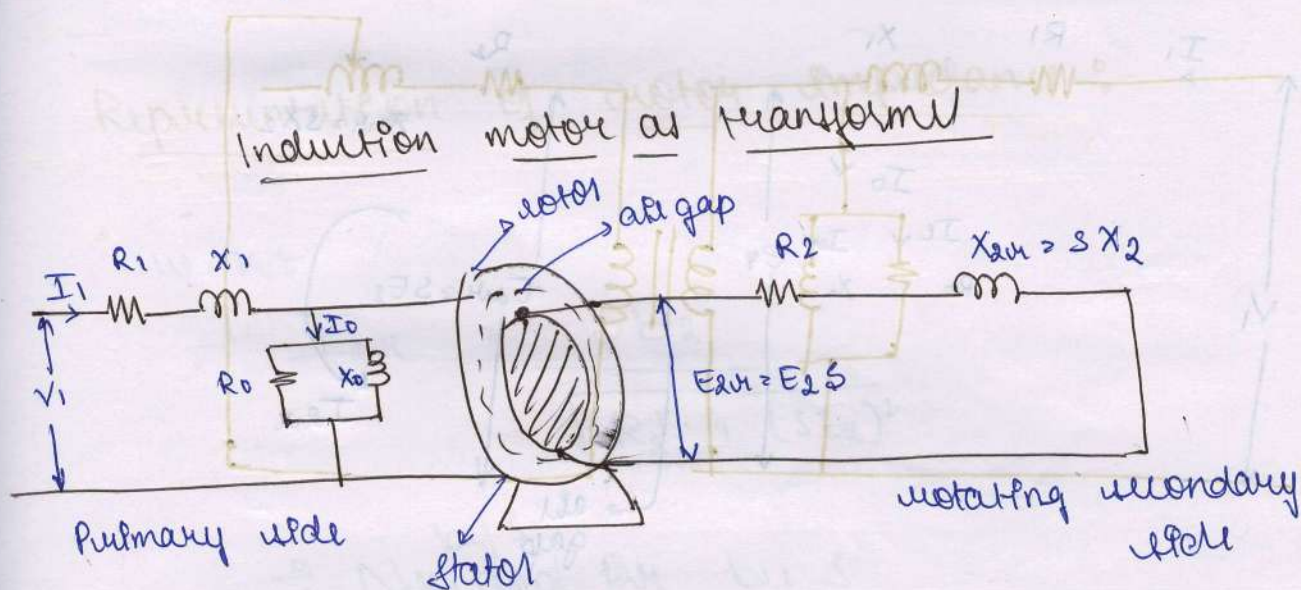
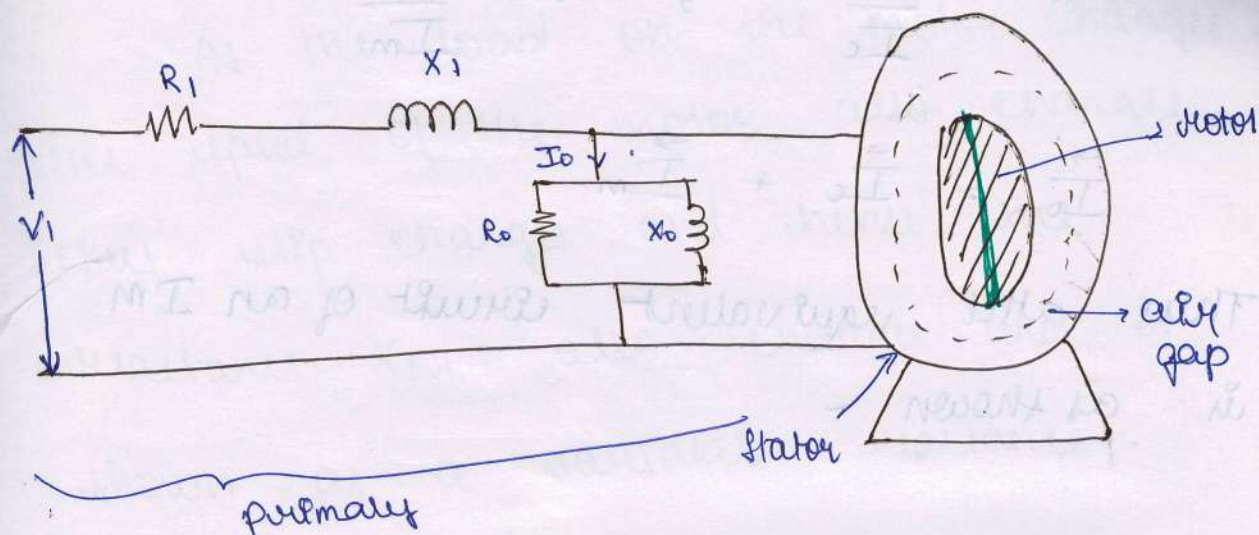
X_{2u} = motor reactance per phase

in running condition.

R_1 = stator resistance per phase

X_1 = stator reactance per phase

Thus the induction motor can be represented as a transformer, as shown.



⇒ Note :-

- When I_m is on no load, it draws current from the supply to produce the flux in air gap and to supply iron losses.

- This current I_0 has two ~~mag~~ components I_m and I_c .

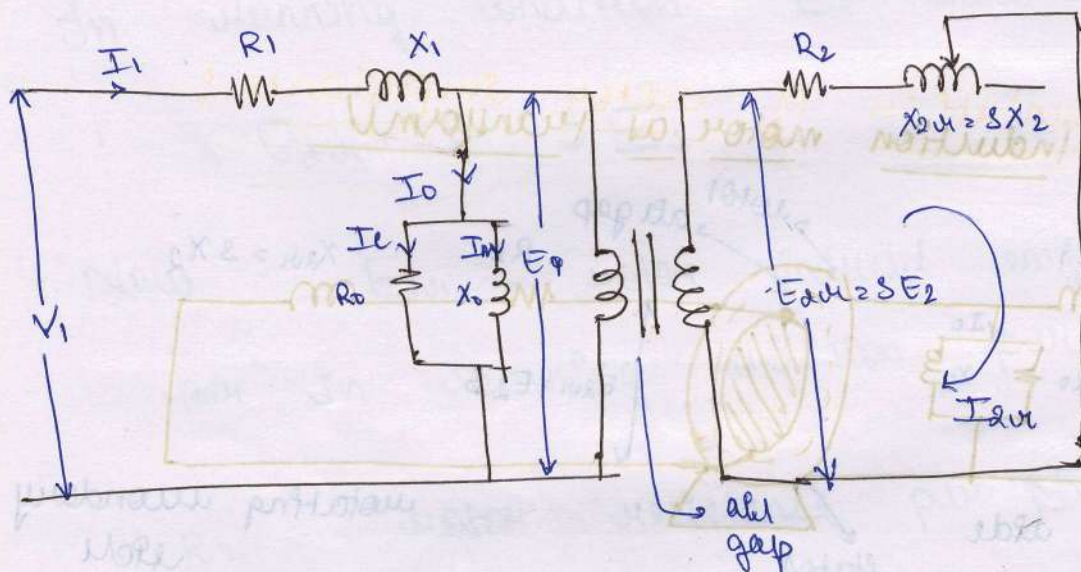
I_m = magnetising component which ~~supply~~ sets up the flux in the core and air gap

I_c = active component which supplies

$$R_0 = \frac{V_1}{I_c} \quad ; \quad X_0 = \frac{V_1}{I_m}$$

$$\vec{I}_0 = \vec{I}_c + \vec{I}_m$$

Thus the equivalent circuit of an IM is as shown -



Basic equivalent circuit

Rotor current in running condition is given by -

$$I_{2r} = \frac{E_{2r}}{Z_{2r}}$$

$$= \frac{E_{2r}}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$= \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{Amp}$$

Note :-

As the load on the motor changes, the speed of the motor also changes, thus slip changes and hence the reactance X_{2M} also changes and is shown as a variable reactance.

Representation of motor impedance :-

w.k.t

$$I_{2M} = \frac{sE_2}{\sqrt{R_2^2 + (sE_2)^2}}$$

$\div$ No. by s and E_2 by s^2

$$I_{2M} = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + \left(\frac{sE_2}{s}\right)^2}}$$

$$I_{2M} = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}}$$

|| Thus it is assumed that the equivalent motor ~~also~~ has a fixed reactance X_2 , fixed voltage E_2 but

a variable resistance $\frac{R_2}{s}$

Now $\frac{R_2}{s}$ can be written as

$$\frac{R_2}{s} = R_2 + \frac{R_2}{s} - R_2$$

$$= R_2 + R_2 \left[\frac{1}{s} - 1 \right]$$

$$\boxed{\frac{R_2}{s} = R_2 + R_2 \left[\frac{1-s}{s} \right]}$$

The variable rotor resistance $\frac{R_2}{s}$

has two parts

- (i) Rotor resistance R_2 itself which represents copper loss
- (ii) $R_2 \left[\frac{1-s}{s} \right]$ which represents the

load resistance R_L which is called electrical equivalent of mechanical load on the motor.

X_{ar} = armature reaction reactance

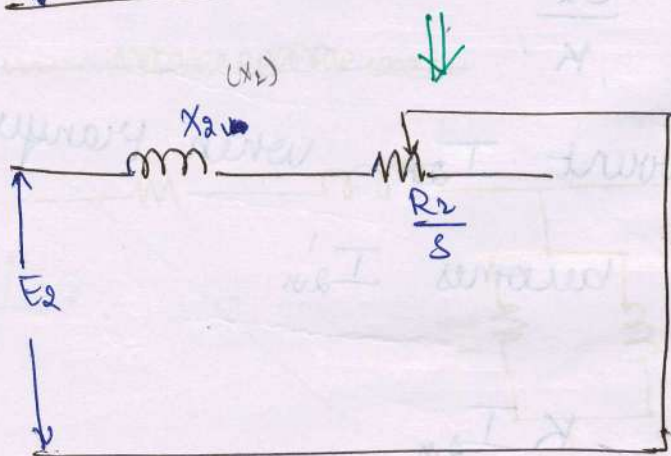
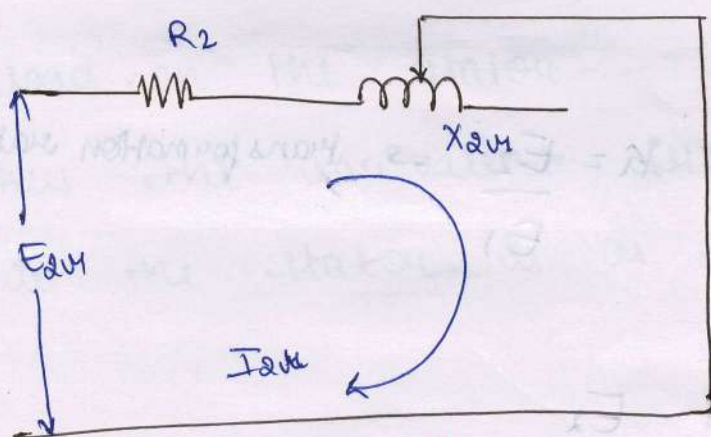
Note :-

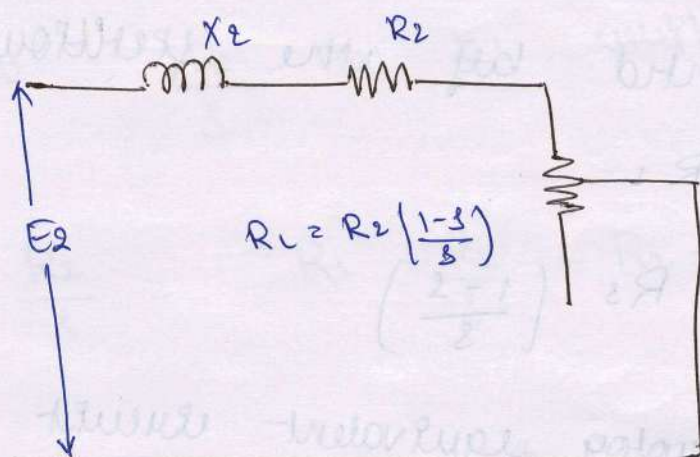
$$R_1 = R_2 \left(\frac{1-s}{s} \right)$$

The mechanical load on the motor is represented by the fictitious resistance R_1

$$R_1 = R_2 \left(\frac{1-s}{s} \right)$$

⇒ Thus the motor equivalent circuit can be shown as





⇒ Equivalent circuit referred to stator :-

Transfer all the motor parameters to the stator

w. K T

$$K = \frac{E_2}{E_1} \rightarrow \text{transformation ratio}$$

$$\therefore E_2' = \frac{E_2}{K}$$

The motor current I_{2u} when transferred to the stator becomes I_{2u}'

$$R_L = R_2 \left(\frac{1-s}{s} \right)$$

$$I_{2u}' = K I_{2u}$$

$$= \frac{K s E_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$R_2' = \frac{R_2}{k^2}$$

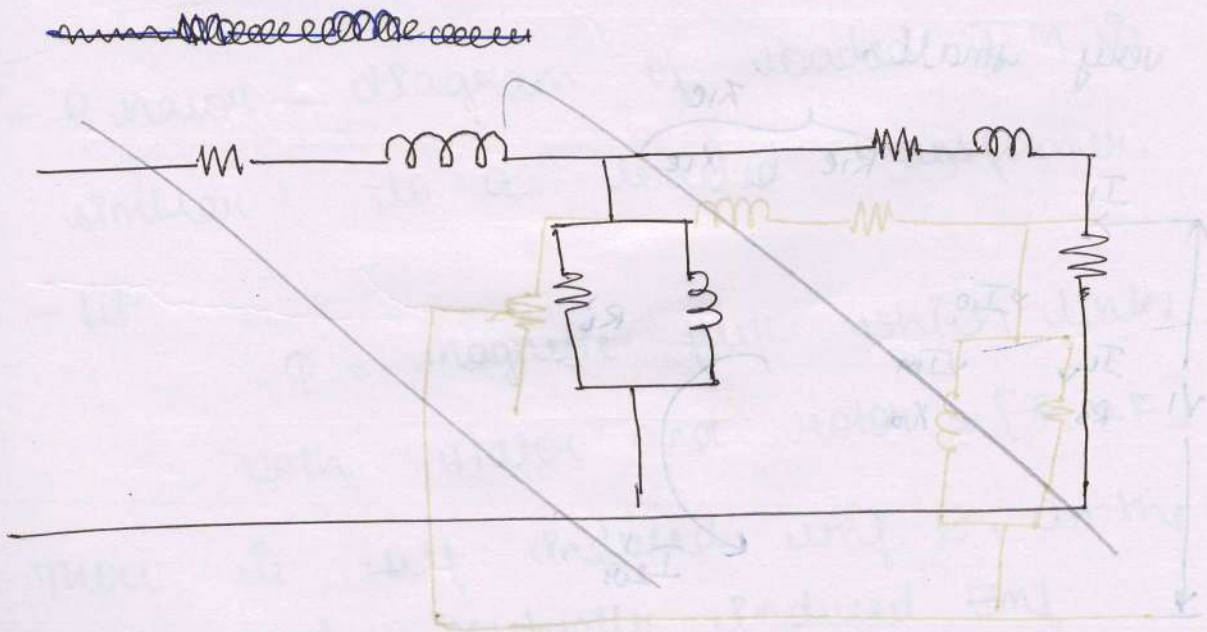
$$X_2' = \frac{X_2}{k^2}$$

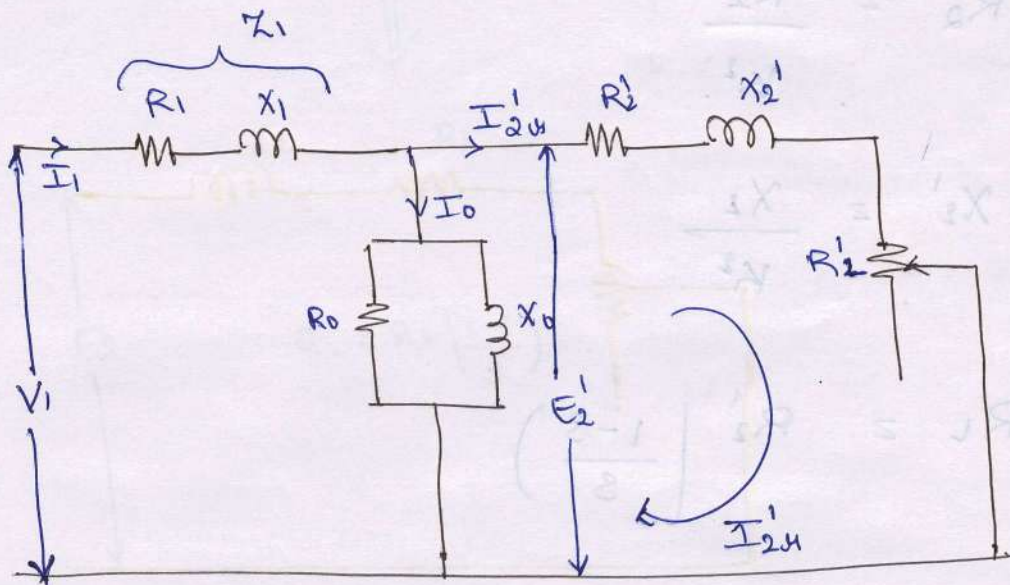
$$R_L = R_2' \left(\frac{1-s}{s} \right)$$

$$R_L = \frac{R_2}{k^2} \left(\frac{1-s}{s} \right)$$

Thus R_L is the required mechanical load on the motor

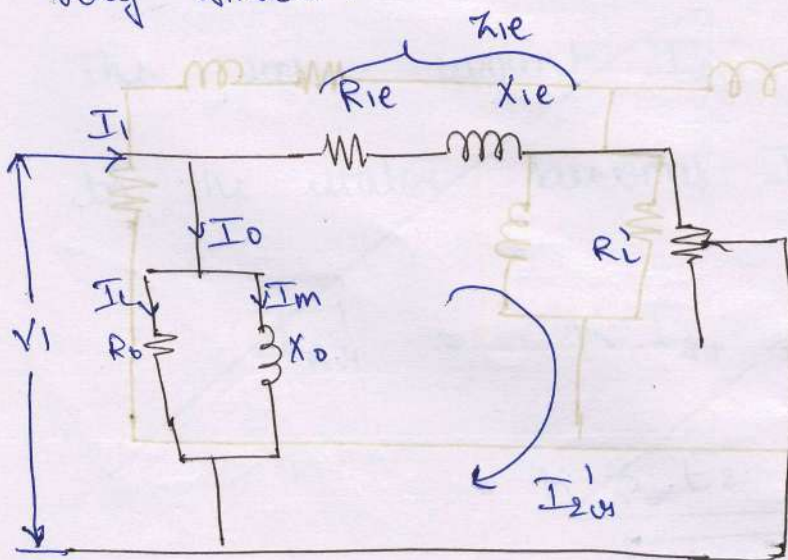
Thus the equivalent circuit referred to the stator is as shown -





Approximate equivalent circuit:-

By shifting the exciting ~~at~~ circuit $[R_0 \text{ and } X_0]$ to the left of R_1 and X_1 , we get the approximate equivalent circuit. Due to this, we are neglecting the drop across R_1 and X_1 due to I_0 which is very small.



R_{1e} = electrical equivalent of mechanical iron losses resistance

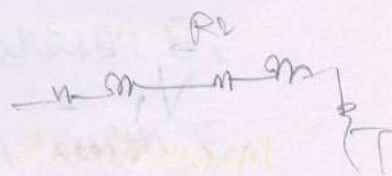
R_{ie} = equivalent resistance referred to stator

$$R_{ie} = R_1 + R_2'$$

X_{ie} = equivalent reactance referred to stator

$$X_{ie} = X_1 + X_2'$$

$$\vec{I}_1 = \vec{I}_0 + \vec{I}_{2a}'$$



where

$$\vec{I}_0 = \vec{I}_c + \vec{I}_m$$

$$Z_{ie} = R_{ie} + j X_{ie}$$

→ Phasor diagram of induction motor :-

- Phasor diagram of loaded I_m is similar to a loaded transformer.

- It

Φ = magnetic flux which links both stator and rotor = [RMF]

There is self induced emf E_1 in the stator and mutually induced emf E_2 in the rotor under running

Let R_1 = stator resistance / phase

X_1 = stator reactance per phase

The supply voltage V_1 has to overcome self induced emf E_1 and also supply voltage drops $I_1 R_1$ and $I_1 X_1$

$$\vec{V}_1 = -\vec{E}_1 + \vec{I}_1 R_1 + \vec{I}_1 X_1$$

$$\vec{V}_1 = -\vec{E}_1 + \vec{I}_1 (R_1 + jX_1)$$

$$\vec{V}_1 = -\vec{E}_1 + \vec{I}_1 Z_1$$

motor induced emf in running condition

has to supply drop across impedance

as the motor is short circuit.

$$\vec{E}_{2s} = \vec{I}_{2s} R_2 + \vec{I}_{2s} X_{2s}$$

$$\vec{E}_{2s} = \vec{I}_{2s} (R_2 + jX_{2s})$$

$$\vec{E}_{2s} = \vec{I}_{2s} Z_{2s}$$

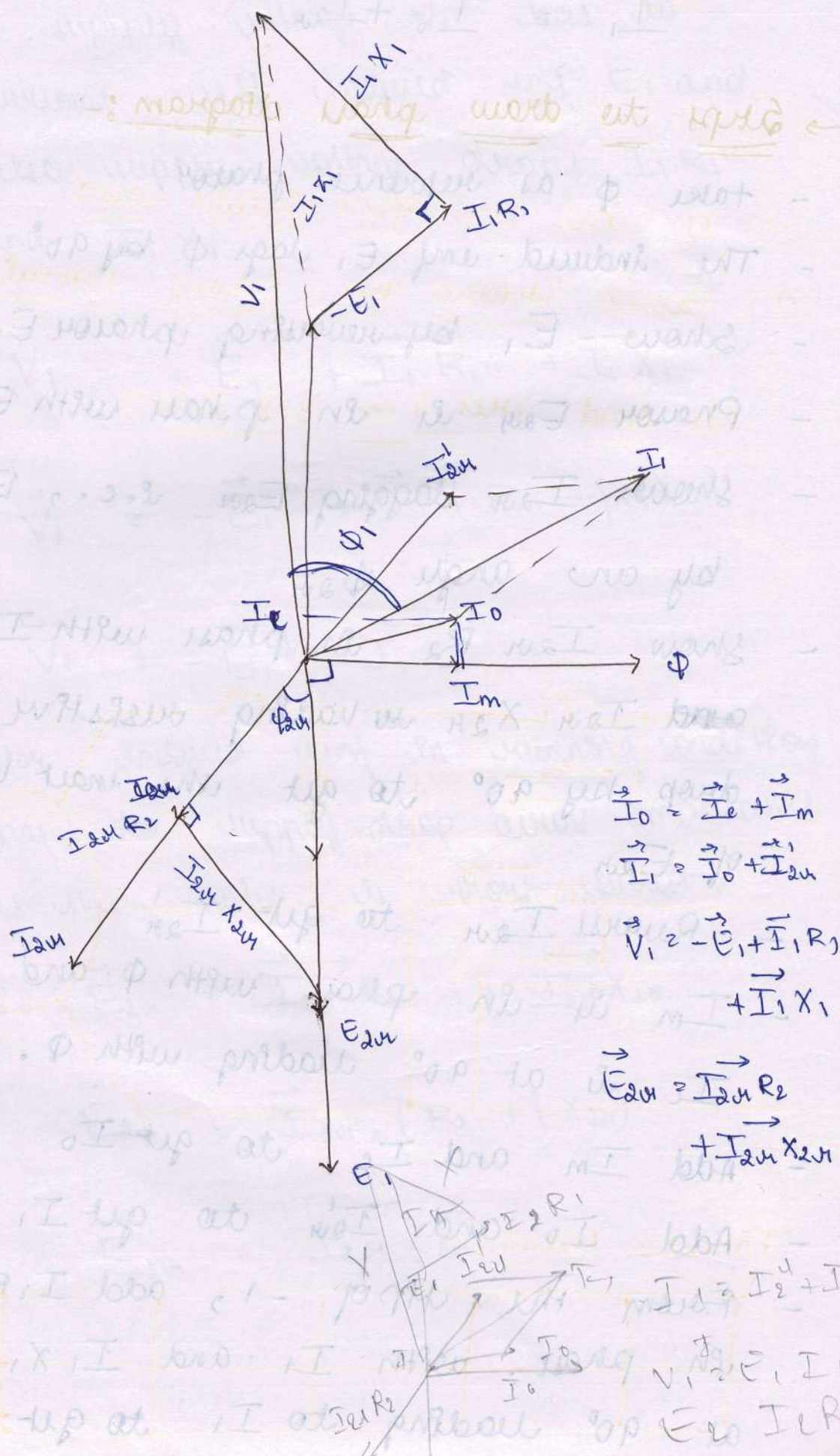
$$I_{2m}' = K I_{2m}$$

$$\vec{I}_1 = \vec{I}_0 + \vec{I}_{2m}'$$

→ Steps to draw phasor diagram:-

- take ϕ as reference phasor
- The induced emf E_1 lags ϕ by 90°
- Show $-E_1$ by reversing phasor E_1
- Phasor E_{2m} is in phase with E_1
- Show I_{2m} lagging E_{2m} i.e., E_1 by an angle ϕ_{2m}
- Show $I_{2m} R_2$ in phase with I_{2m} and $I_{2m} X_{2m}$ leading resistance drop by 90° to get the exact location of E_{2m}
- Reverse I_{2m} to get I_{2m}'
- I_m is in phase with ϕ and I_c is at 90° leading with ϕ .
- Add I_m and I_c to get I_0
- Add I_0 and I_{2m}' to get I_1
- From the tip of $-I_1$, add $I_1 R_1$ in phase with I_1 and $I_1 X_1$ at 90° leading to I_1 to get

- Angle b/w V_1 and I_1 is ϕ_1



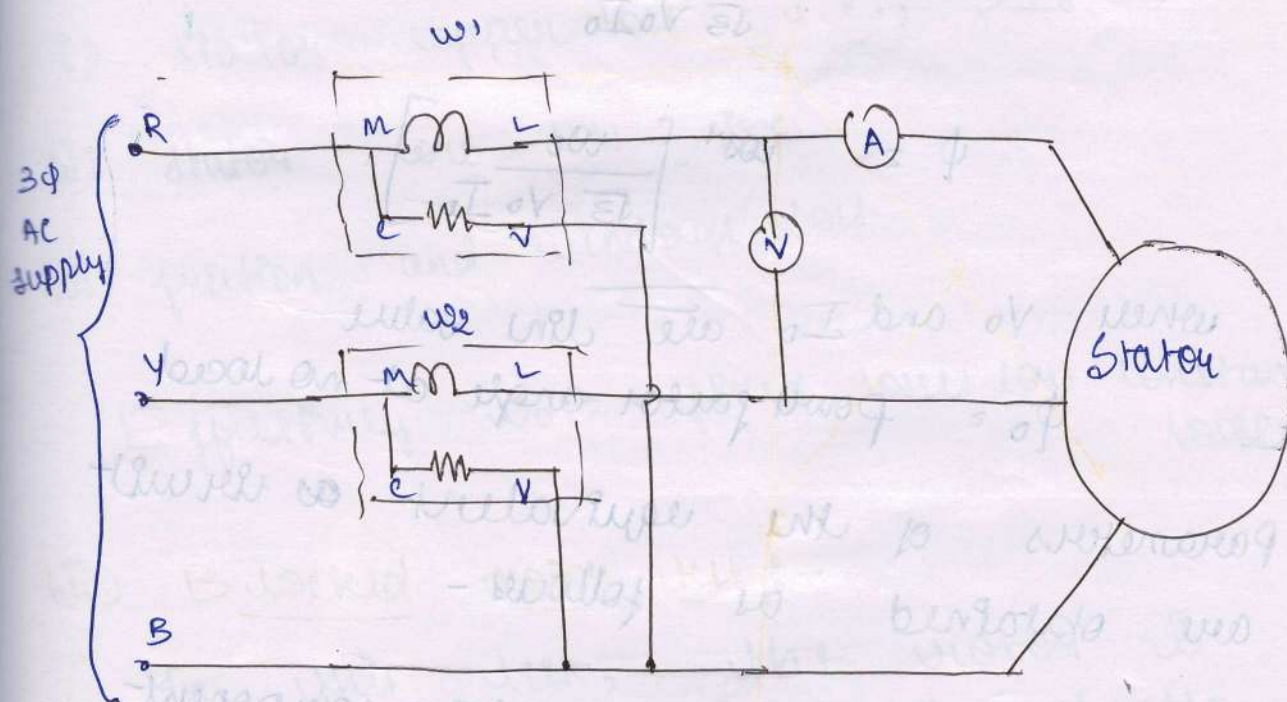
→ Circle diagram of a 3 ϕ IM :-

- To plot the circle diagram of 3 ϕ IM, two tests are conducted

(i) No load (or) open circuit test :-

(ii) Block motor (or) Short-circuit test :-

(i) No load test on 3 ϕ IM :-



Tabular column :-

Sr. No.	V_0 (volts)	I_0 (amps)	W_1 (watts)	W_2 (watts)	$W_0 = W_1 + W_2$ (watts)
1)	400				

If pf is low $\rightarrow$ reactive power is more
 means $\downarrow$

on no load is very low

$\hookrightarrow I_m$ draws reactive power

$\hookrightarrow$ wattmeter measures only active power

Wattmeter drawback

interchange (and V)

$\hookrightarrow$ so L.P.F wattmeter is used
 $\hookrightarrow$ on no load pf is low

calculations are :-

1) No load power

$$W_0 = \sqrt{3} V_0 I_0 \cos \phi_0 \text{ watts}$$

$$\cos \phi_0 = \frac{W_0}{\sqrt{3} V_0 I_0}$$

$$\phi = \cos^{-1} \left[\frac{W_0}{\sqrt{3} V_0 I_0} \right]$$

where V_0 and I_0 are line value

$\phi_0 =$ power factor angle at no load

Parameters of the equivalent circuit are obtained as follows -

$I_e = I_0 \cos \phi_0 \rightarrow$ active component of the no load current

$I_m = I_0 \sin \phi_0 \rightarrow$ reactive component of the (or) magnetizing component of no load current.

$$R_0 = \frac{V_0}{I_e}$$

$$= \frac{V_0}{I_0 \cos \phi_0}$$

$$X_0 = \frac{V_0}{I_m}$$

$$= \frac{V_0}{I_0 \sin \phi_0}$$

The no load input w_0 consists of

- i) stator copper loss i.e., $3 I_0^2 R_1$
- ii) stator core loss w_{sc}
- iii) friction and windage loss

Effectively $w_0 = \text{fixed losses (or) constant losses}$

(ii) Blocked motor test:-

In this test, the motor is blocked manually and is not allowed to rotate.

Then $s = 1$ (maximum)

$$R'_1 = R'_2 \left(\frac{1-s}{s} \right)$$

$$\therefore R_1' = R_2' \quad \left(\frac{1}{s} \right) \quad \text{at } s = 1$$

$$\underline{\underline{R_1' = R_2'}}$$

If This test is similar to short-circuit test on a transformer when supply is switched on and rated current is passed ~~it~~ through the IM at a reduced voltage.

Let $V_{sc} =$ ~~voltage~~ short circuit reduced voltage (line value)

$I_{sc} =$ short-circuit current (or) rated current (line value)

$W_{sc} =$ short circuit input power

$$W_s = \sqrt{3} V_{sc} I_{sc} \cos \phi_{sc} \text{ watts}$$

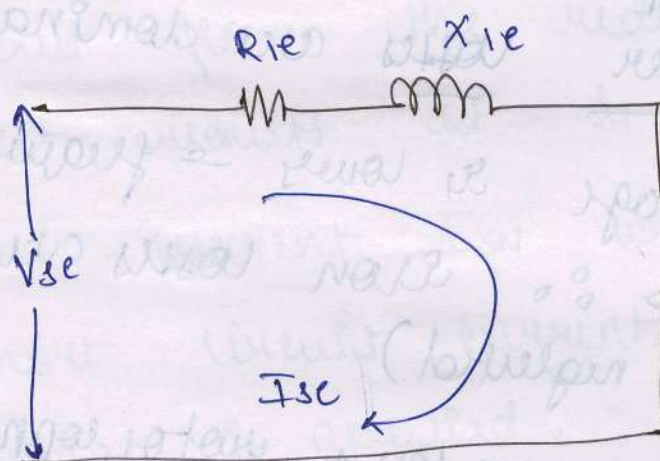
$$\cos \phi_{sc} = \frac{W_{sc}}{\sqrt{3} V_{sc} I_{sc}}$$

$$\therefore \phi_{sc} = \cos^{-1} \left[\frac{W_{sc}}{\sqrt{3} V_{sc} I_{sc}} \right]$$

$\cos \phi_{sc}$ is short circuit power factor angle

Sl. No.	V_{sc} (volts)	I_{sc} (amps)	W_1 (watts)	W_2 (watts)	$W_{sc} = W_1 + W_2$
		Rated			

equivalent circuit is as shown -



$$W_{sc} = 3 I_{sc}^2 R_{ie}$$

$$R_{ie} = \frac{W_{sc}}{3 I_{sc}^2} \quad \Omega / \text{ph}$$

[R_{ie} consumes power]

R_{ie} = equivalent resistance referred to stator

$$Z_{ie} = \sqrt{R_{ie}^2 + X_{ie}^2}$$

$$X_{ie} = \sqrt{Z_{ie}^2 - R_{ie}^2}$$

Note

During this test the stator carries rated current and thus stator copper losses are dominant.

Similarly motor also carries short circuit component and thus motor copper losses are dominant.

↳ Applied voltage is low $\rightarrow$ fluxes are low $\rightarrow$ iron losses are low (to neglected)

$$W_{sc} = \text{stator copper loss} + \text{motor copper loss}$$

(variable losses)

I_{sc} $\rightarrow$ short circuit current at reduced voltage

I_{sn} $\rightarrow$ full rated current at

8/5/2017

It is necessary to obtain short circuit current when normal rated voltage is applied to the motor, this is practically not possible.

But from the reduced voltage test results it is possible to find current I_{SN} which is short circuit current when normal voltage is applied.

∴

V_L = normal rated voltage (line value)

V_{sc} = reduced short circuit voltage

$$\text{then } I_{SN} = I_{sc} \times \left(\frac{V_L}{V_{sc}} \right)$$

Now power input is proportional to the square of the ~~the~~ current

$$W_{SN} = \left(\frac{I_{SN}}{I_{sc}} \right)^2 \times W_{SN} \text{ watts}$$

$$W_{SN} = \text{core losses} + \text{stator copper loss}$$

$$+ \text{Rotor copper loss}$$

→ Construction of circle diagram:-

By using

the data obtained from no load and blocked diagram

circle diagram can be constructed using following steps -

1) Take reference phasor V as reference axis

2) Select suitable current scale such that the diameter of the circle is 20 - 30 cm

From no load test I_0 and ϕ_0 are obtained

3) Draw vector I_0 , lagging V by an angle ϕ_0 . This is line

00

$$W_{SN} \times \left(\frac{V}{I_0} \right) = W_{SN}$$

- 4) Draw horizontal through extremity of I_o i.e., at O' parallel to n axis
- 5) Draw the current ~~I_m~~ I_{sn} calculated from I_{sc} with the same scale lagging V by an angle ϕ_{sc} from the origin O

This is phase $O'A$

- 6) Join $O'A$ called the output line

- 7) Draw a $\perp$ bisector of $O'A$ extended to meet the line drawn $\parallel$ to x axis at point e . This is the centre of the circuit

- 8) with e as centre radius $= O'e$ draw the semi circle which meets the horizontal line ~~p~~ p drawn $\parallel$ to x axis at point B

- 9) Draw a $\perp$ from point A on the n axis to be $O'B$ at point F and D the n axis at point B

- 10) Construction of torque line.

~~construction~~
- construction of torque line

- length AD represents the power input at short circuit W_{sc}

- Now FD represents fixed losses

∴ length of (AF) ∝ (stator cu loss + Rotor cu loss)

Then point E can be located as follows

(i) slip using IM

In slip using IM stator and rotor currents can be measured using ammeters

If I_1 = stator current

I_2 = rotor current

$K = \frac{I_1}{I_2}$ - transformation ratio

∴ Assuming O'E represents the torque line

$$\text{Now } \frac{AF}{EF} = \frac{\text{rotor cu loss}}{\text{stator cu loss}} = \frac{I_2^2 R_2}{I_1^2 R_1}$$

$$= \frac{R_2}{R_1} \left(\frac{I_2}{I_1} \right)^2 = \frac{R_2}{R_1} \times \frac{1}{K^2}$$

$$= \frac{R_2'}{R_1} \quad \left[R_2' = R_2 / K^2 \right]$$

Thus point E is obtained by dividing

the line in the ratio R_2' / R_1

(ii) Salient cage IM:-

$$\text{stator cu losses} = 3 I_{sN}^2 R_1$$

where I_{sN} is phase value

neglecting core loss

$$W_{sN} = \text{stator cu loss} + \text{Rotor cu loss}$$

∴ Rotor cu loss

$$= W_{sN} - \text{stator cu loss}$$

$$= W_{sN} - 3 I_{sN}^2 R_1$$

$$\therefore \frac{AE}{EF} = \frac{W_{sN} - 3 I_{sN}^2 R_1}{3 I_{sN}^2 R_1}$$

Power scale:-

$$\text{Power scale} = \frac{W_{sN}}{A(AD)} \quad \text{watts/cm}$$

Auditing performance parameters using

circle diagram:-

Let the motor take a current of

OP

Using the power scale and various distances, various performance

parameters can be obtained

- i) Total motor i/p $P_T \times \text{power scale}$
- ii) Fixed loss $= S_T \times \text{power scale}$
- iii) Stator cu loss $= R_S \times \text{power loss}$
- iv) Rotor cu loss $= R_R \times \text{power loss}$
- v) Total loss $= Q_T \times \text{power scale}$
- vi) Rotor o/p $= P_Q \times \text{power scale}$
- vii) Rotor p/p $= (P_Q + Q_R) \times \text{power scale}$
- viii) Slip $s = \frac{\text{rotor cu loss}}{\text{rotor p/p}}$
- ix) power factor $= \cos \phi = \frac{P_T}{Q_P}$
- x) motor efficiency $= \frac{P_Q}{P_T}$
- xi) Rotor efficiency $= \frac{P_Q}{P_R}$

→ Full load condition -

The full load motor o/p is given on the name plate of the IM in watt or HP

Calculate the difference corresponding to full load output using the power scale.

Extend AD ^{from A} up ~~to~~ upwards, & draw
 the distance corresponding to
 full load o/p i.e., A'

Draw parallel to the o/p line OA,
 from A' to meet the circle at
 point P

Q1) Draw a circle diagram for a 20 HP,
 50 Hz, 3 ϕ star connected IM with
 the following data

(i) No load test:- 400 V, 9 A, 0.2 p.f lag

(ii) Blocked motor test:- 200 V, 50 A; 0.4 p.f lag

Determine the line current, efficiency
 and slip for full load condition from the
 circle diagram.

Soln

$$\cos \phi_0 = 0.2$$

$$\phi_0 = 78.46^\circ, I_0 = 9 \text{ A}, \text{ ~~VS 2400V~~}$$

$$\cos \phi_{sc} = 0.4$$

$$\phi_{sc} = 66.42^\circ, I_{sc} = 50 \text{ A}, \text{ ~~VS 200V~~}$$

$$I_{SN} = \frac{I_{sc} \times V_L}{V_{sc}}$$

$$= \frac{50 \times 400}{200}$$

$$\underline{I_{SN} = 100A}$$

$$W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \phi_{sc}$$

$$= \sqrt{3} \times 200 \times 50 \times 0.4$$

$$= 6928.20 W$$

$$W_{SN} = \cancel{W_{sc}} \cdot W_{sc} \left(\frac{I_{SN}}{I_{sc}} \right)^2$$

$$= \cancel{6928.20} \cdot 6928.20 \left(\frac{100}{50} \right)^2$$

$$\underline{W_{SN} = 27712.81 \text{ watts}}$$

(or)

$$W_{SN} = \sqrt{3} \times \cancel{V_L} V_L I_{SN} \cos \phi_{sc}$$

$$= \sqrt{3} \times 400 \times 100 \times 0.4$$

$$= \underline{27712.81 \text{ watts}}$$

$$\underline{27712.81}$$

$$\text{Power scale} = \frac{W_{SN}}{A(\text{AD})}$$

$$= \frac{327712.81x}{8.6}$$

$$= \underline{\underline{3222.41 \text{ watts/cm}}}$$

$$\text{Full load o/p} = 20 \text{ HP} = 20 \times 735.5$$

$$= 14710 \text{ W}$$

$$= 14710$$

$$= \underline{\underline{4.56 \text{ cm}}} = A'$$

$$A(\text{OP}) = 6.5 \text{ cm} \times 5 = \underline{\underline{32.5 \text{ A}}}$$

= full load line current

$$P_f = \text{power factor} = \cos(\text{angle angle by OP w.r.t y axis})$$

$$= \cos(29)$$

$$= \underline{\underline{0.8746}} \text{ lag}$$

$$\eta = \frac{P_O}{P_S} = \frac{407}{557} \times 100$$

$$= \underline{\underline{82.54}}$$

$$\text{Slip} = \frac{QR}{PR} = \frac{0.65}{5.03} \times 100$$

$$= \underline{\underline{12.22\%}} = \underline{\underline{0.1222}}$$

~~To determine various maximum quantities from generalized circle diagram~~

To determine various maximum quantities from generalized circle diagram:-

(i) Max O/P :-

Draw a line // to $O'A$ which is also tangent to the circle at point M.

The max O/P is given by $\angle(MN)$ at power scale.

(ii) max η :-

It occurs on the highest point on the circle i.e., point 1

max p/p is given by

$\cos(\phi - \phi')$ at power scale.

(iii) max torque % -

Draw a line parallel to the torque line which is also tangent to the circle at point J.

$\cos(\phi - \phi')$ represents the max torque in synchronous watts at the power scale.

(iii) max power factor:-

Draw a line tangent to the circle from the origin O meeting the circle at point H.

Drop a $\perp$ from H on the axis until point I.

For maximum pf = $\cos \angle OHI$

$$(iv) \frac{HI}{OI}$$

Q1) Draw the circle diagram for from no load and short circuit test of a 3ϕ ~~14.92~~ 14.92 kW, 400 V, 6 pole IM with the following test data (line value).

No load :- 400 V, 11 A, 0.2 pf (lag)

SC test :- 100 V, 25 A, 0.4 pf (lag)

The motor cu loss at standstill is half the total cu losses.

From the diagram find - line current, slip, efficiency, power factor at full load, max torque.

Soln

$$\cos \phi_0 = 0.2$$

$$\phi_0 = 78.46^\circ$$

$$I_0 = 11 \text{ A}$$

$$\cos \phi_{sc} = 0.4$$

$$\phi_{sc} = 66.42^\circ$$

$$I_{sc} = 25 \text{ A}$$

$$I_{SN} = \frac{I_{sc} \times V_L}{V_{sc}}$$

$$= 100 A$$

$$W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \phi_s$$

$$= \sqrt{3} \times 100 \times 25 \times 0.8$$

$$= 1732.05 W$$

$$W_{SN} = \sqrt{3} V_L I_{SN} \cos \phi_{sc}$$

$$= \sqrt{3} \times 400 \times 100 \times 0.8$$

$$= 27712.81 W$$

$$\text{Power scale} = \frac{W_{SN}}{1(A.D)} = \frac{27712.81}{8.4}$$

$$= 3299.14 \text{ watt/cm}$$

$$\text{Full load o/p} = 14.92 \text{ kW}$$

$$\lambda (AA) = \frac{\text{Full load o/p}}{\text{Power sent}}$$

$$= \frac{14.92 \times 10^3}{3299.14}$$

$$= 4.52 \text{ W}$$

$$\lambda (OP) = 6.445$$

$$P_e, V_{OP} = \underline{32A}, \text{ W} \text{ (2)}$$

$$P_f = \text{W} (32)$$

$$= 0.8480 \text{ lag} \text{ (i)}$$

$$P_e, V_{OP} = \underline{32A}, \text{ W} \text{ (ii)}$$

$$\text{W} \text{ slip} = \frac{Q_R}{P_R}$$

$$= \frac{0.25}{5.2}$$

$$= 0.048 = 4.80\% \text{ (i)}$$

$$P_e, V_{OP} = \underline{32A}, \text{ W} \text{ (ii)}$$

$$\text{W} \text{ slip} = \frac{Q_R}{P_R}$$

$$= \frac{0.25}{5.2}$$

$$\eta = \frac{P_R}{P_T}$$

$$= \frac{4.45}{5.4} \times 100$$

$$= \underline{82.40\%}$$

$$u(JK) = 8.5$$

$$= 8.5 \times 3299.14$$

$$= 28.042069$$

$$= \cancel{28.042}$$

$$= 28.042 \text{ Syn watts}$$

→ [synchronous watts]

Q2) 50 kW, 6 poles, 50 Hz, 450 V, 3 ϕ

Im gave the following test results

- (i) No load test :- 450 V, 20 A, 0.15 Pf lag
- (ii) SC test :- 200 V, 150 A, 0.3 Pf lag

The ratio of stator to rotor cu loss

is 5 : 4. Draw a circle diagram

and determine

- (i) full load current and Pf
- (ii) max torque and max power i/p
- (iii) slip at full load
- (iv) Efficiency on full load

$$\cos \phi_0 = 0.955$$

$$\phi_0 = \cancel{60}^\circ 81.37^\circ$$

$$\cos \phi_{sc} = 0.3$$

$$\phi_{sc} = 72.054$$

$$I_{sN} = \frac{I_{sc} \times V_L}{V_{sc}}$$

$$= \frac{150 \times 450}{200}$$

$$= 337.5 \text{ A}$$

$$W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \phi_1$$

$$= \sqrt{3} \times 200 \times 150 \times \cancel{0.955} \cdot 0.3$$

$$= 15,588.45 \text{ W}$$

$$W_{sN} = \sqrt{3} V_L I_{sN} \times 0.3$$

$$= \underline{\underline{78916.56 \text{ W}}}$$

$$\text{power scale} = \frac{W_{sN}}{4(\text{AD})} = \frac{78916.56}{4.85}$$

$$= \underline{\underline{16271.45 \text{ W/km}}}$$

Full load o/p = 50 kW

$I(AA') \rightarrow$

$$\frac{\text{Stator Cu loss}}{\text{Rotor Cu loss}} = \frac{\frac{5}{4} \cdot \frac{4}{5} \cdot \frac{5}{4}}{\frac{5}{4}} = \frac{EF}{AE}$$

$$10.25$$

$$10.25 AE = EF$$

$$AF = AE + EF$$

$$AF = AE + 10.25 AE$$

$$= 20.25 AE$$

(or)

$$AE = \frac{AF}{20.25}$$

$$= \frac{4065}{20.25}$$

$$= 200.66 \text{ cm}$$

$$= 200.66 \text{ cm}$$

$$\text{Power scale} = \frac{50 \times 10^3}{16271.45}$$

$$= 3077 \text{ W}$$

$$L(OP) = 30.6 \mu m$$

$L(OP) \neq$ turn seal

$$= 30.6 \times 50$$

r

$$= 1.8 A$$

$$P_f = - \cos(18)$$

$$L(OP) = \frac{1}{2} \phi$$

$$\cos \phi = 1$$

man torque

$$JK = 5.6 \mu m$$

or on sign

$$= 103.98 \times 10^3$$

man p p

$$\Rightarrow L - L = 6.9$$

$L(LL')$ Power side

$$= \text{man P ID } 128.12 Kw$$

sup

$$\frac{Q P}{P V}$$

$$= 3.44 \%$$

$$P = 90.3$$



MODULE - 5

SYNCHRONOUS

MOTORS

alternator $\rightarrow$ field winding / rotor winding

$\rightarrow$ given DC supply $\rightarrow$ single phase

$\rightarrow$ mechanical to electrical energy conversion

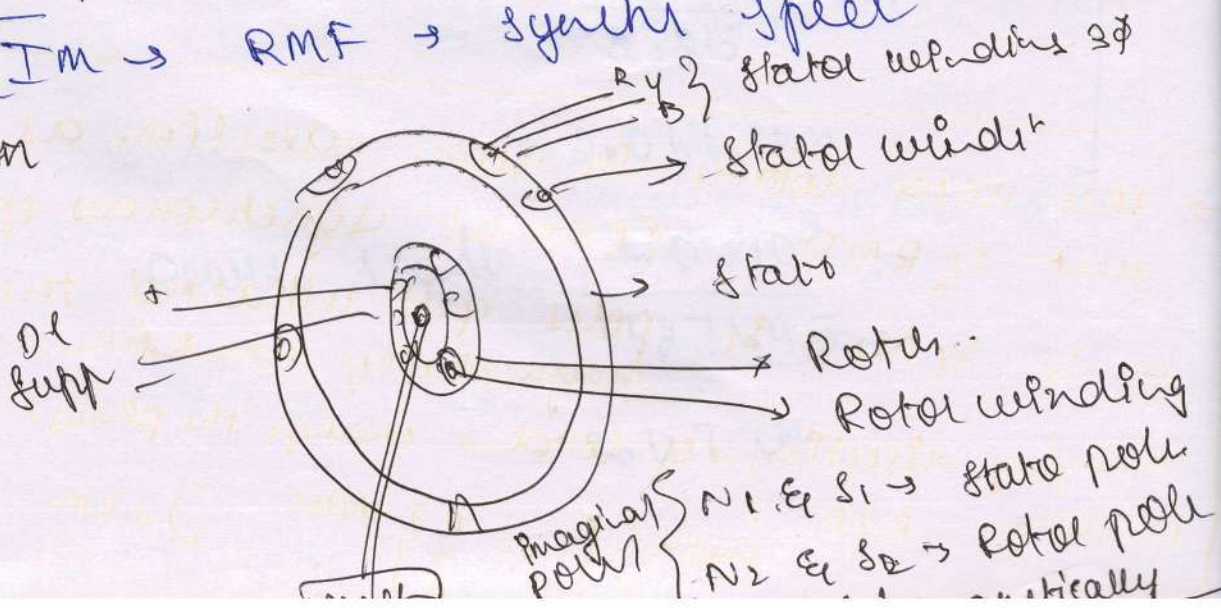
synchronous motor (or) generator remains same
 $\rightarrow$ working on the principle of magnetic locking

locking

Not self starting

[IM $\rightarrow$ RMF $\rightarrow$ synch speed

con



→ construction of synchronous motor - 3ϕ

Two major parts are -
 stator and rotor

used motor -

Salient pole motor
 Non salient pole motor

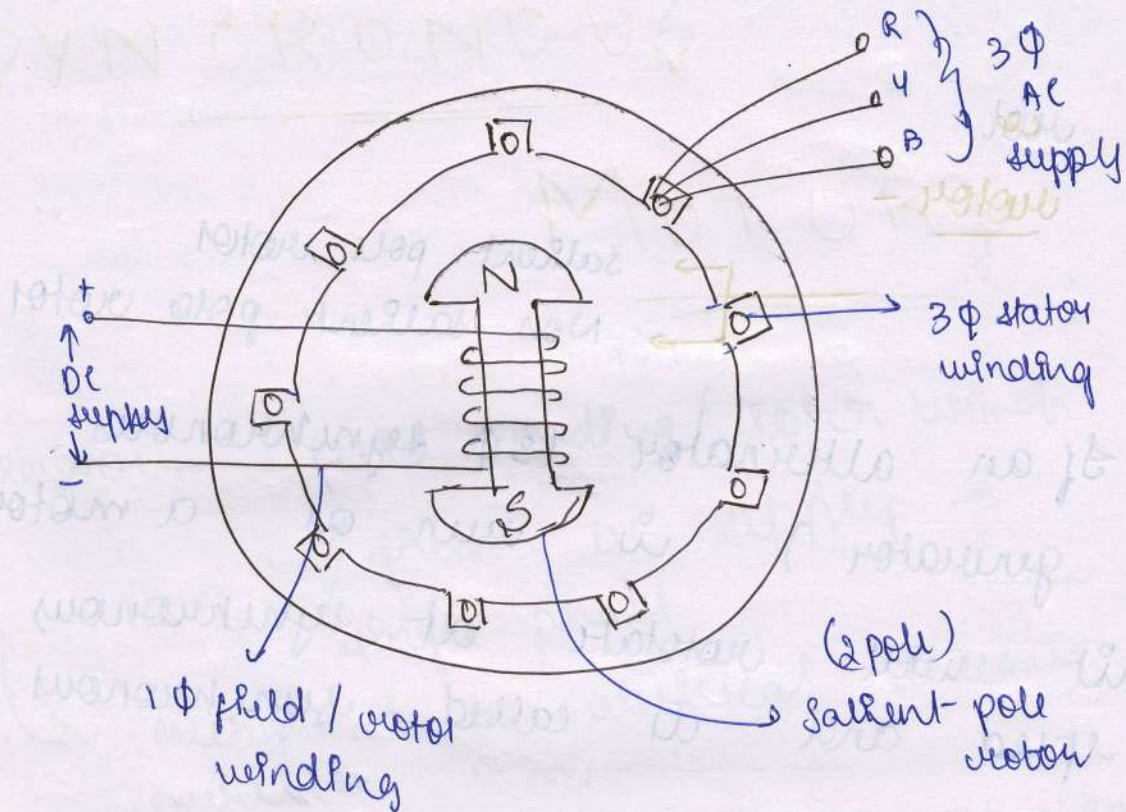
If an alternator (3ϕ synchronous generator) is run as a motor, it will rotate at synchronous speed and is called synchronous motor.

The speed at which RMP rotates is given by -

$$N_s = \frac{120f}{P} \text{ rpm}$$

→ Then rotor rotates in same direction as that of RMP (RMP at synchronous speed)
 $N_1 - S_2$ & $N_2 - S_1$ poles get attracted. This magnetically locks them.
 Changing direction of rotor → change the phase sequence RYB supply $\left\{ \begin{matrix} R \\ Y \\ B \end{matrix} \right\} \Rightarrow \left\{ \begin{matrix} R \\ Y \\ B \end{matrix} \right\}$ stator $\rightarrow$ R.M.

→ Schematic representation of 3 ϕ synchronous motors :-



90% motor are salient pole motor.

It contains 1 ϕ field winding $\rightarrow$

DC supply using slip rings

(DC supply $\rightarrow$ station) $\rightarrow$ rotor (rotating)

$\rightarrow$ through slip ring)

stator $\rightarrow$ is supplied by 3 ϕ

AC supply and it produces RMF

→ Working principle :-

- Works on the principle of magnetic locking
- To have magnetic locking there must exist two unlike poles and the magnetic axis of one two poles must be brought very close to each other.

- Assume the stator is wound for 2 poles (say) with 50 Hz supply.

- Let these two poles stator poles be N_1 and S_1 which are rotating at a speed of N_s (rpm)

- Assume the direction of rotation of RMF as clockwise

- Assume Then when the field / rotor winding

is excited by a DC supply,

it also produces two poles,

- Let these two motor poles be N_2 and S_2

At standstill N_1, S_1 are rotating
at N_1 rpm

N_2, S_2 are ~~not~~ yet
if by some how methods N_1 get attracted
to S_2 and N_2 get attracted
to S_1 then magnetic locking
takes place
and N_2, S_2 starts rotating at
synchronous speed.

At start/beginning when mechanical
load is applied momentarily it
get reduced and still maintains
synchronous speed

Synchronous motor will never rotate

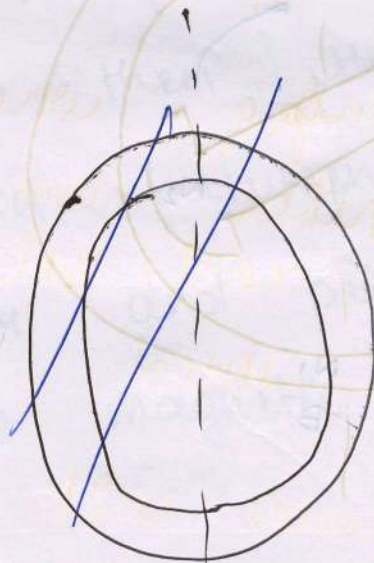
in any other speed other than
synchronous. (It is effect of RMF
that is considered as 2 pole N_1, S_1)

⇒ Synchronous motor is not self
starting.

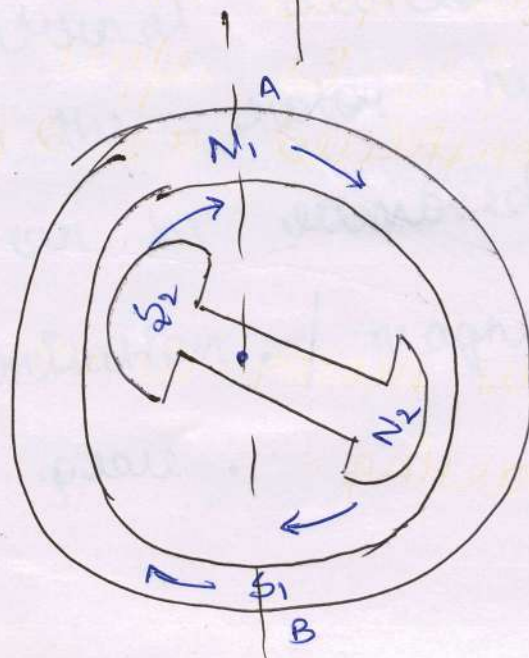
consider the RMF is equal
between N_1 and S_1

and are at a position where
 stator magnetic axis is vertical
 i.e., along A-B

At this instant, the rotor
 poles are arbitrarily positioned, at
 this instant motor is stationary
 and ~~unlike~~ unlike poles will try
 to attract each other.

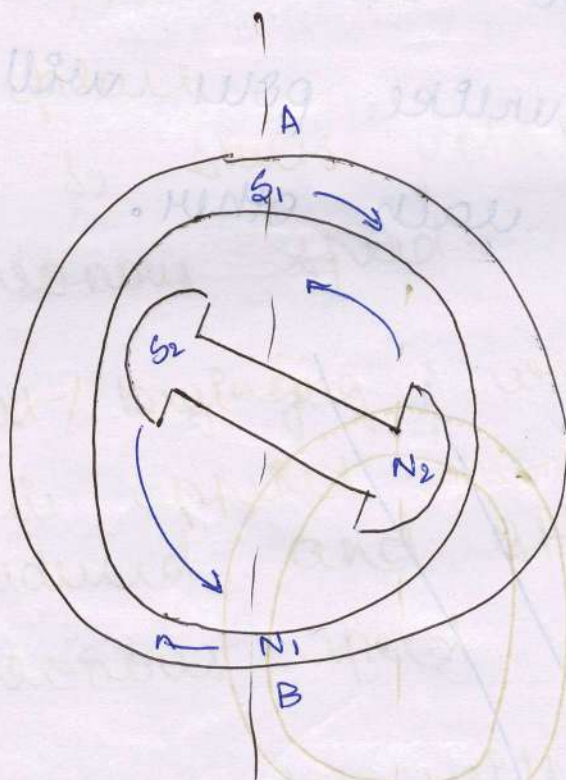


case (ii) :-



Due to this the motor gets subjected to an instantaneous torque in clock wise direction.

Case (ii): -



→ To start a synchronous motor:-

Suppose the motor is rotated by some external means at a speed almost equal to synchronous speed and then motor is excited to produce the poles.

At a certain instance, the stator and motor poles will fall each other and then there is a force of attraction between the stator and motor and pulls both of them into magnetic locking condition.

The external device used to rotate the motor may synchronous speed can be removed once synchronisation / magnetic locking takes place.

→ Methods of starting synchronous motor :-

- 1) using pony motors :-
- 2) using damper winding :-
- 3) As a slip ring IM :-
- 4) using small DC motor-machine :-

1) Using pony motors :-

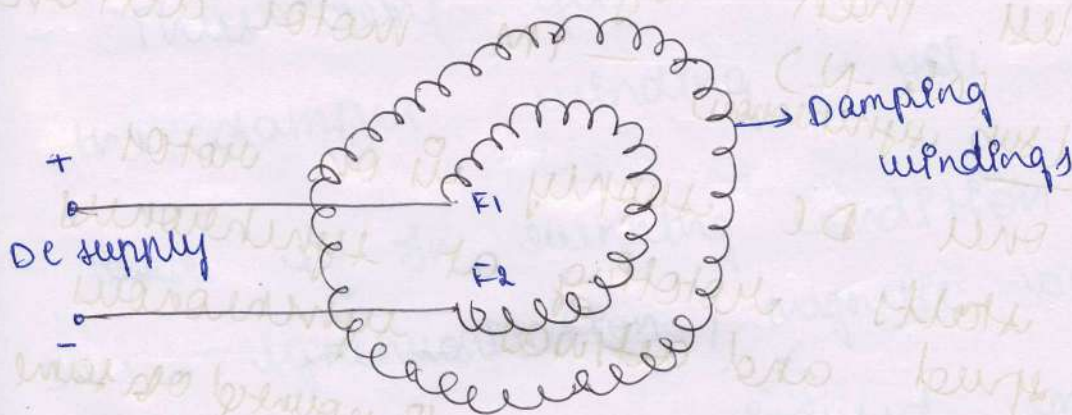
In this method, the motor of synchronous motor is brought to synchronous speed with the help of a small internal IM called pony motor.

Once synchronisation is established, the pony motor is decoupled / removed.

2) Using damper winding:-

In this method, in addition to the normal field winding (or rotor winding), an additional winding consisting of m bars are placed in the slots in the pole face.

The bars are short circuited with the help of end rings. Such an additional winding is called damper winding.



Damped $\rightarrow$ pole phase lock

the synchronous motor works as

IM Initially (At start

- supply - $3\phi \rightarrow$ stator $\rightarrow$ ~~RMF~~ $\rightarrow$

RMF $\rightarrow$ it produces and cuts
motor windings (DC is ~~not~~ supplied)

and this RMF $\rightarrow$ current is
produced $\rightarrow$ emf is induced $\rightarrow$
from it produced $\rightarrow$ Ind damper
winding

i) flux $\rightarrow$ (i) RMF
(ii) Damp winding flux

Thus due to interaction of 2 fluxes

motor starts rotating at speed

less than synchronous speed

($N_s \rightarrow N$)
(sub synchronous) - IM motor action

once DC supply is on motor

starts rotating at synchronous

speed and actual synchronous
motor (is mounted on same
motor)

$\hookrightarrow$ for damper winding is rotating

in same synchronous speed
so there is change in

At start total in station
Damp winding in station
RME in rotating
b

Same $N_2 - S_1$, $N_1 - S_2$ magnetic
locking — synchronous speed
is so that does not stop
the IM motor action stops and
synchronous action continues.

→ No flux is induced —

— At start, the RME acts the
stationary damper winding which
is placed in the rotor slots.

— Thus emf gets induced in
the damper winding — this

— ~~once~~ In running condition,
once synchronisation / magnetic locking
takes place, the induced emf
in the damper winding becomes
zero because after magnetic locking
the motor will rotate at synchronous

speed N_s which is same as RMF.

[Dumper windings are placed on the rotor in the winding slots]

[Dumper winding operation remains same initially as that of squirrel cage IM] (or) short circuited rotor]

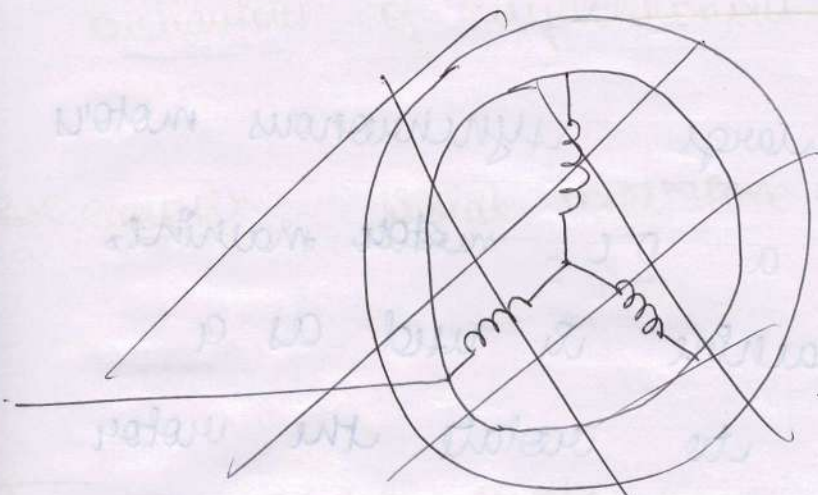
- starter has to be connected to start

3) As a slip ring IM -

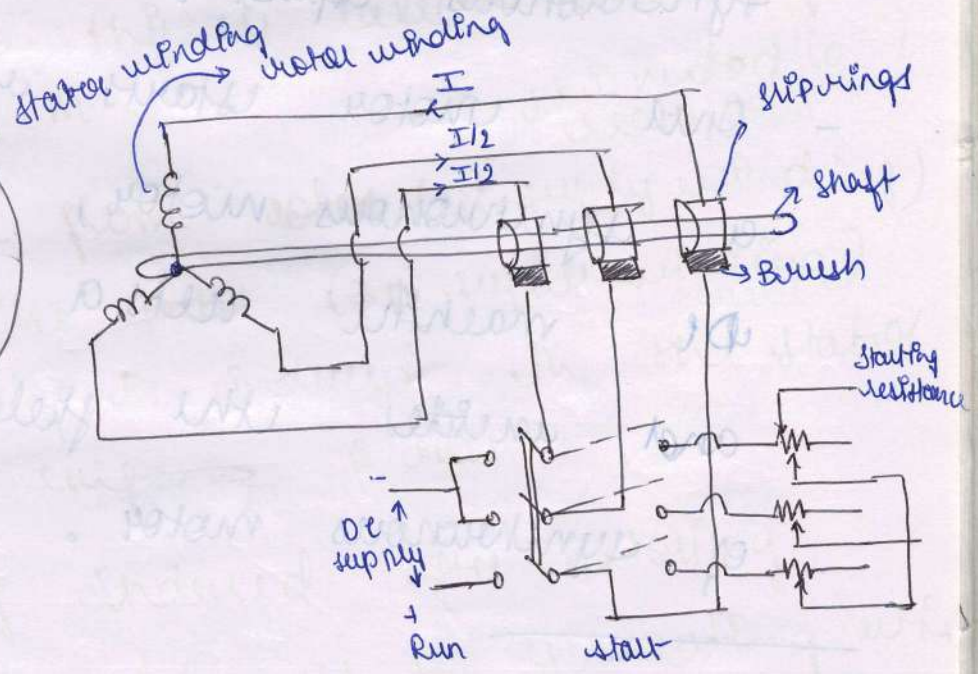
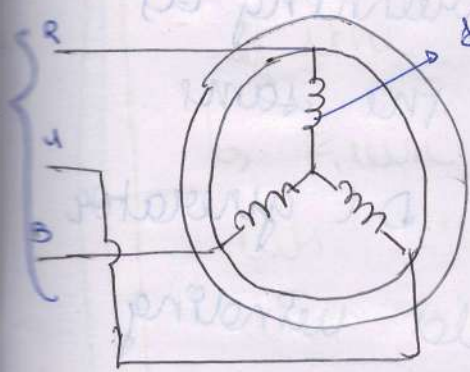
- In the dumper winding method it is difficult to achieve high starting torque.

- To achieve high torque instead of shorting the dumper winding, it is designed to form a 3 ϕ star or delta connected winding

- The 3 ends of this winding are brought out through slip rings



AC supply



At start
 TPDT in start position
 It (i) It acts as slip ring IM
 Here starting torque can be varied by
 by varying starting resistance
 (ii)
 // same steps as previous one // ϵ
 but slip ring omitted to variable
 resistance to get high starting

4) Using small DC machine:-

- Usually large synchronous motors we use a DC ~~motor~~ machine.
- This machine is used as a DC motor to rotate the rotor of the synchronous motor at synchronous speed.

- Once motor starts running as a synchronous motor, the same DC machine acts as a DC generator and excites the field winding of synchronous motor.

Once synchronous motor attains synchronous speed it can be used as synchronous generator.

⇒ Behaviour of synchronous motor on loading

Example :- ideal condition on no load :-

In 3ϕ AC motor

emf induced in stator $\rightarrow$ back emf $\rightarrow$ this is called back emf of the IM -

$\hookrightarrow$ Flux from rotor (DC supply winding) cuts the stator (3ϕ supply winding) and emf is induced in the stator $\rightarrow$ Back emf $\rightarrow$ ϕ this $\nearrow$ it is needed to create poles

This emf induced after motoring of motor i.e., as a generating machine on the stator winding is called back emf of 3ϕ IM;

consider a DC shunt motor whose voltage eqn is given by

$$V = (E_b) + I_a R_a$$

(8/4) n/m

$$E_b = V - I_a R_a$$

$$E_b = \frac{P \phi N Z}{60 A}$$

E_b = back emf i.e., induced emf in a dc motor due to generating action

Similarly in synchronous motor, there is an induced emf in the 3 ϕ stator winding due to alternator action (synchronous generator)

after magnetic locking takes place

This induced emf is known as back emf of a synchronous motor whose magnitude is given

$$E_{b, syn} = K_e K_d \phi T_p \text{ volts}$$

where $K_e = \frac{2\pi}{60}$

$$K_d = \frac{2\pi n (m p / 2)}{m \omega n (p / 2)}$$

$$\therefore \boxed{E_{bph} \propto \phi}$$

As the speed is always synchronous (Ns) frequency is constant and hence magnitude of back emf can be controlled by flux ϕ produced by the motor.

Let R_a = stator winding resistance
and X_s = synchronous reactance of the stator

then synchronous impedance of the motor is given by -

$$\underline{Z_s = R_a + jX_s} \quad \Omega / \text{ph}$$

Thus similar to a DC motor, voltage equation for a synchronous motor can be written as

$$\underline{\vec{V}_{ph} = \vec{E}_{bph} + \vec{I}_{aph} \vec{Z}_s}$$

$$\boxed{\vec{I}_{apn} = \frac{\vec{V}_{pn} - \vec{E}_{bph}}{Z_s}} \quad \text{Amps}$$

• $\rightarrow$ (i) Ideal condition on no load:-

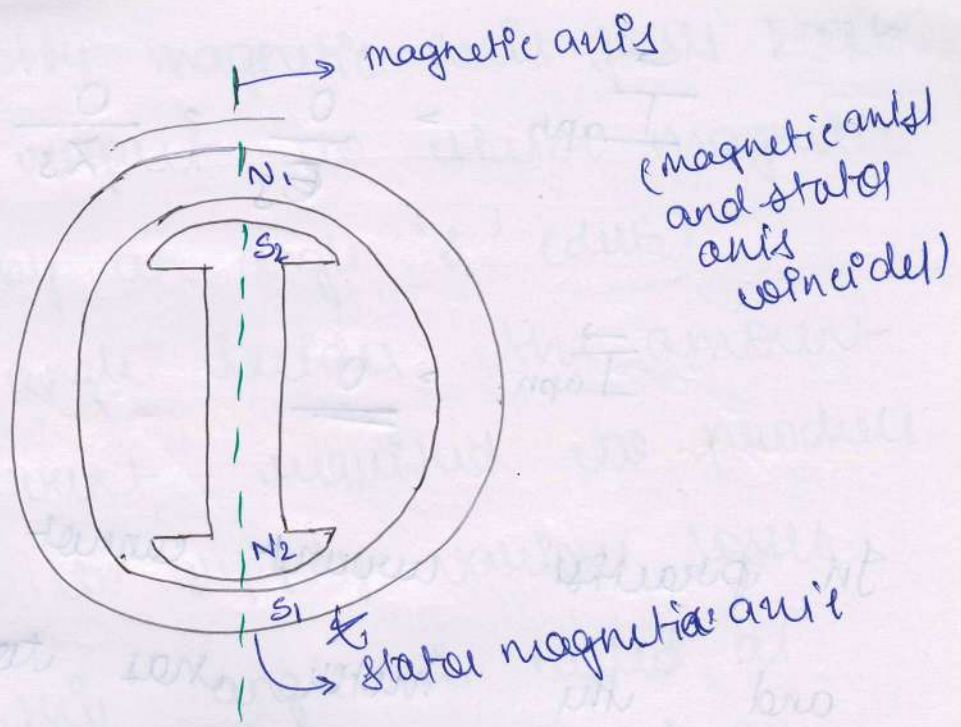
magnitude of E_{bph} is made almost equal to V_{pn} by controlling the flux produced by field winding

Thus for ideal condition on no load, neglecting various losses

$$\boxed{V_{pn} = E_{bph}}$$

Under this condition the magnetic locking b/w stator & rotor is in such a way that the magnetic axis of both coincide with each other





(a) ideal condition with no load

The magnitudes of ~~E_{bph}~~ E_{bph} and V_{ph} are equal but they are in opposite direction Lenz's law.



$$|E_{bph}| = |V_{ph}|$$

~~W.K.T~~
W.K.T

$$\vec{I}_{ap} = \frac{\vec{V}_{ph} - \vec{E}_{bph}}{Z_s}$$

$$\vec{I}_{aph} = \frac{0}{E_s} = \frac{0}{Z_s}$$

$$\vec{I}_{aph} = \underline{\underline{0}}$$

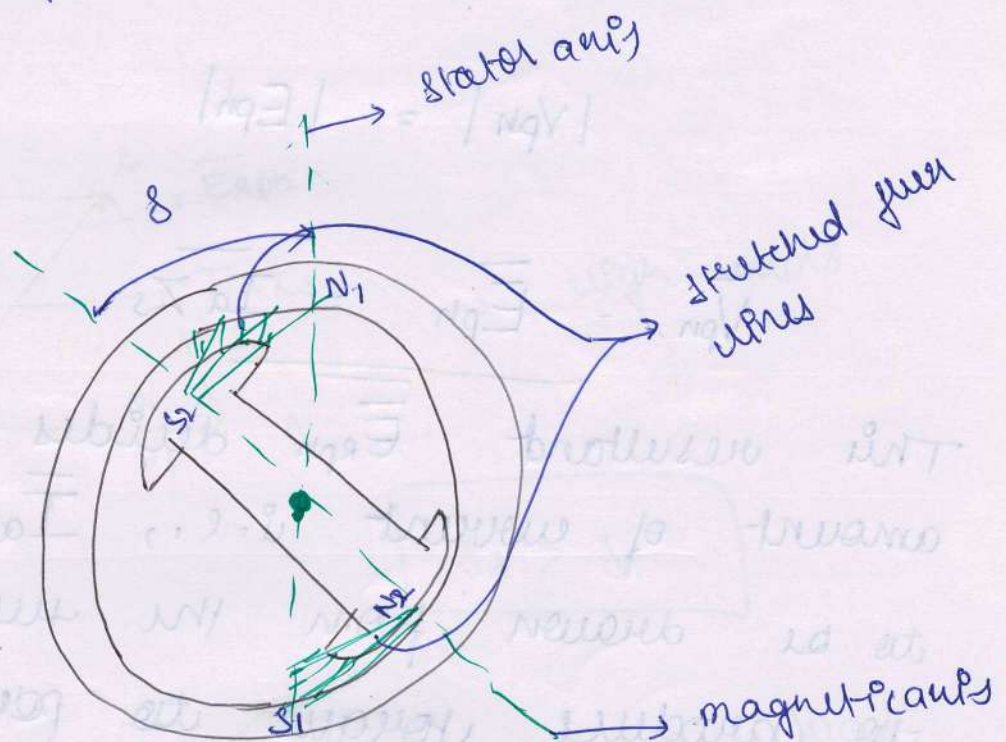
In practice current cannot be zero and the motor has to supply mechanical and even losses under no load condition

To produce the torque required to overcome these losses - the motor draws current from the supply.

• $\rightarrow$ (iii) On no load (with losses):-

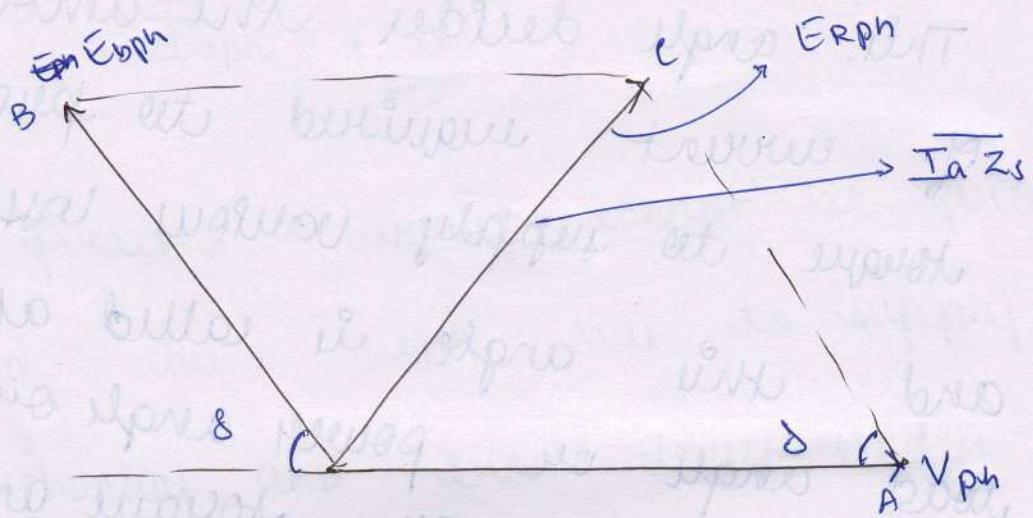
Due to various losses present practically in a motor on no load, the magnetic locking exists between the stator and motor but in such a way that ^{there} exists a small angular difference b/w the ~~ex~~ axis of 2 magnetic fields.

- The rotor magnetic axis falls back with respect to stator magnetic axis by an angle ' δ ' (delta)
- This angle decides the amount of current required to produce torque to supply various losses and this angle is called as load angle or power angle or coupling angle or torque angle or angle of retardation.



greater angle (δ)
 (above $\delta = 90^\circ$)
 speed is stopped

magnetic locking is less
 and synchronous



phasor diagram for no load with losses

$$|V_{ph}| = |E_{ph}|$$

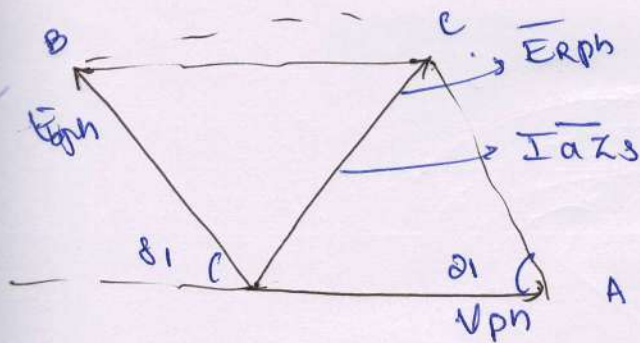
$$\overline{V_{ph}} - \overline{E_{ph}} = \overline{I_a Z_s} = \overline{E_{Rph}}$$

This resultant $\overline{E_{Rph}}$ decides the amount of current i.e., $\overline{I_{aph}}$ to be drawn from the supply to produce torque to ~~run~~ which meets the various losses under no load condition & is very small and hence $\overline{E_{Rph}}$ is also very small.

1 $\rightarrow$ (iii) Synchronous motor on load

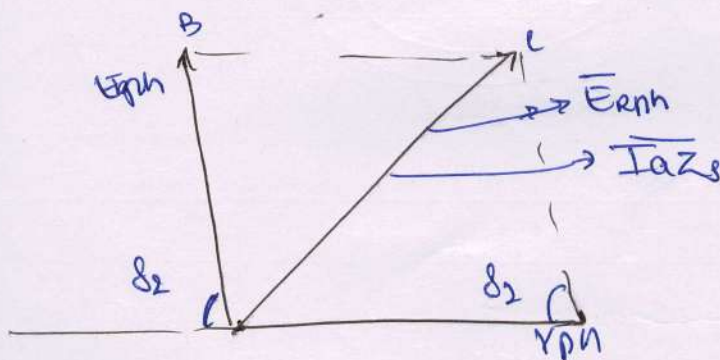
As the load on the synchronous motor increases there is no change in its speed but load angle δ gets affected

Hence if the load increases, δ increases but speed remains synchronous



$\rightarrow$ light load

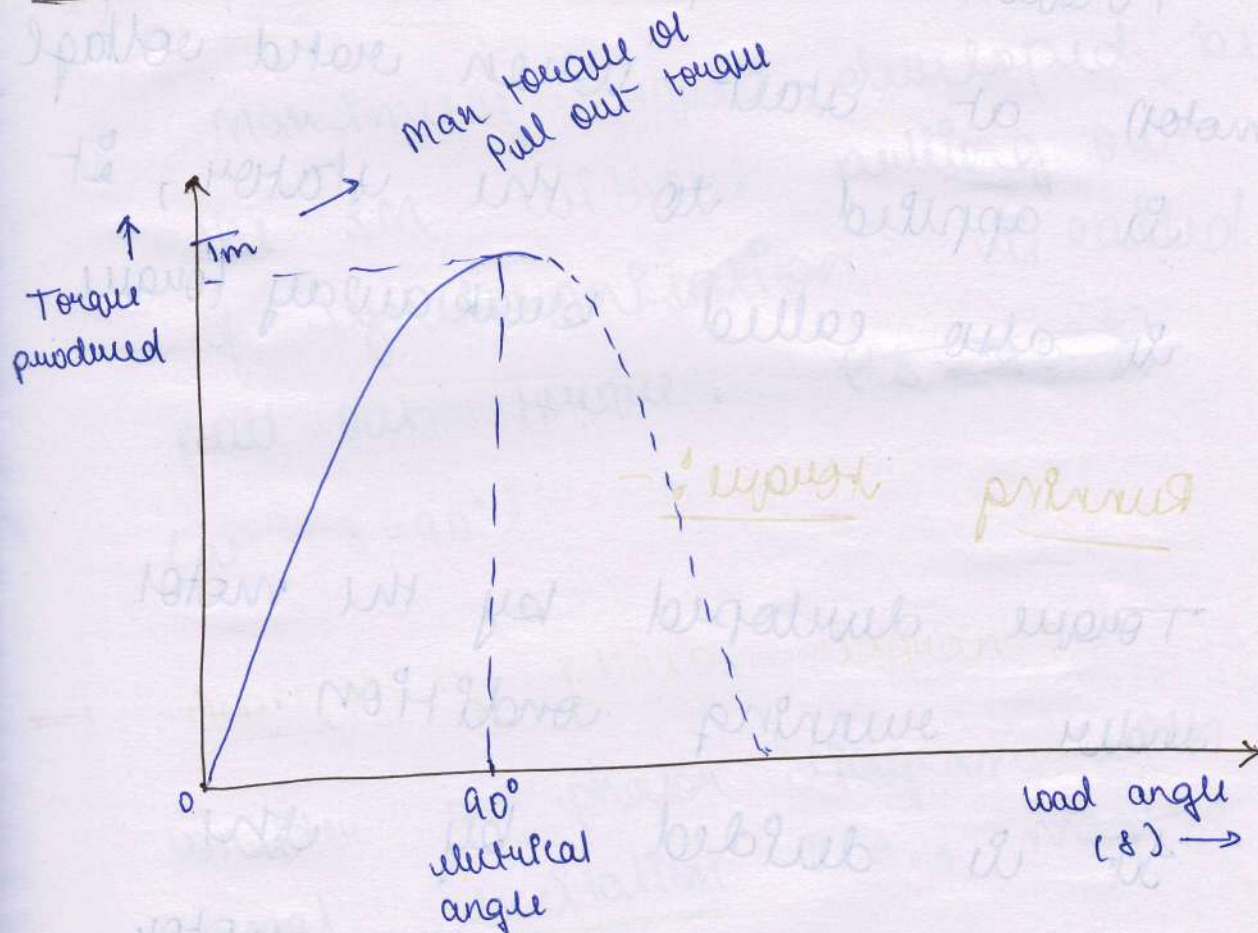
$$\delta_2 > \delta_1$$



Heavy load

MODULE - 5 [continuation]

Torque angle characteristics :-



as load on synch motor increases
& also increases
beyond 90° magnetic locking is
lost and torque is reduced, and
motor ~~stop~~ comes out of synchronisation
and no torque is not produced

→ Types of torques in a synchronous motor :-

1) Starting torque :-

Torque developed by SM (synchronous motor) at start when rated voltage is applied to the stator, it is also called break away torque.

2) Running torque :-

Torque developed by the motor under running condition.

It is decided by the output rating of the motor and speed.

3) Pulling torque :-

When the speed of the motor is near synchronous speed.

The amount of torque developed by the motor at the time of pulling into synchronisation is called ~~synchron~~ pulling torque.

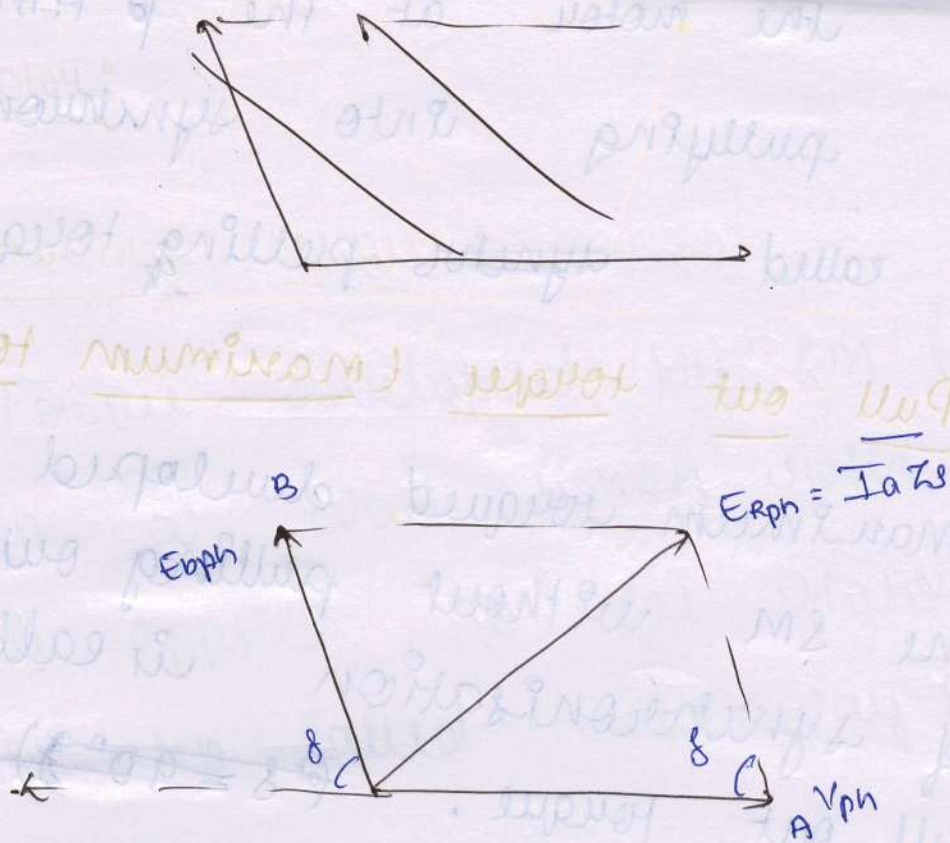
4) Pull out torque (maximum torque)% -
 maximum torque developed by the SM without pulling out of synchronisation is called pull out torque. ~~(at $\delta = 90^\circ$)~~
 (at $\delta = 90^\circ$)

⇒ Analysis of phasor diagram:-

consider a phasor diagram with normal excitation i.e., ~~not~~
 ~~$|E_{bph}|$~~ ~~is~~ ~~$|V_{ph}|$~~

$$|E_{bph}| = |V_{ph}|$$

$$|E_{bph}| = |V_{ph}|$$



$$\rightarrow \vec{V}_{ph} - \vec{E}_{ph} = \vec{I}_a \vec{Z}_s$$

$$\Rightarrow \vec{V}_p - \vec{E}_{ph} = \vec{I}_a \vec{Z}_s$$

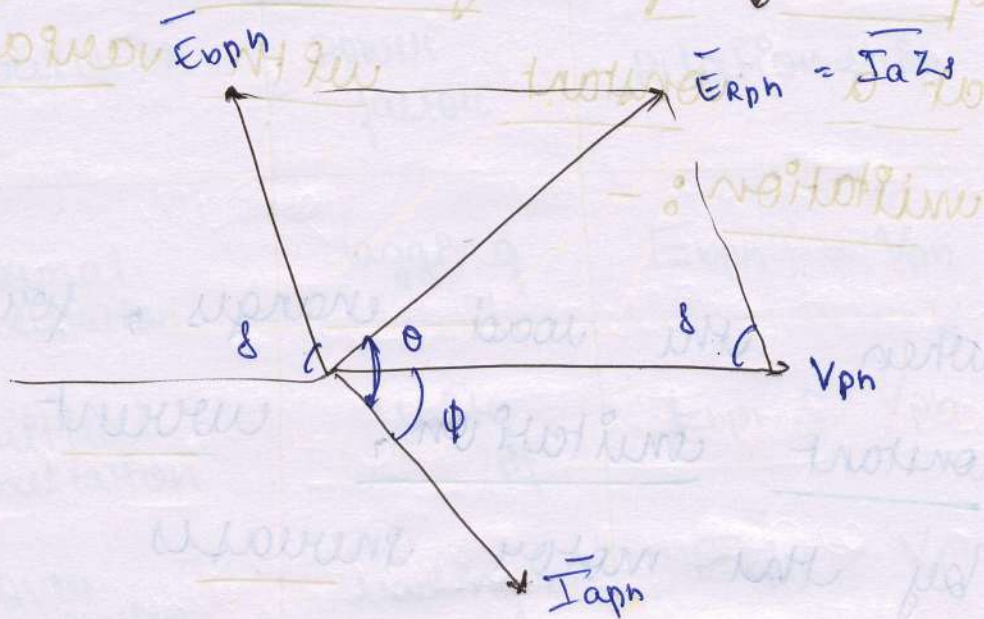
$$Z_s = R_a + j X_s \quad \Omega / ph$$

$R_a \rightarrow$ stator resistance per phase

$X_s \rightarrow$ synchronous reactance of stator per phase

$$|Z_s| = \sqrt{R_a^2 + X_s^2}$$

phasor diagram under normal working condition



$$\theta = \tan^{-1} \left(\frac{X_s}{R_a} \right)$$

The angle θ is called impedance angle or internal machine angle.

Practically R_a is very small compared to X_a

X_a = reactance of stator (Not sync.)

Hence θ tends to 90°

operation of synchronous motor
at a constant with variable
excitation :-

when the load changes, for a
constant excitation, current drawn
by the motor increases

But if the excitation (field
current I_f) is changed keeping
the load constant, SM reacts
by changing its power factor
of operation.

Types of excitation :-

- 1) Normal excitation
- 2) under excitation
- 3) over excitation
- 4) critical excitation



Sd.
No.

1)

2)

3)

4)

23/2

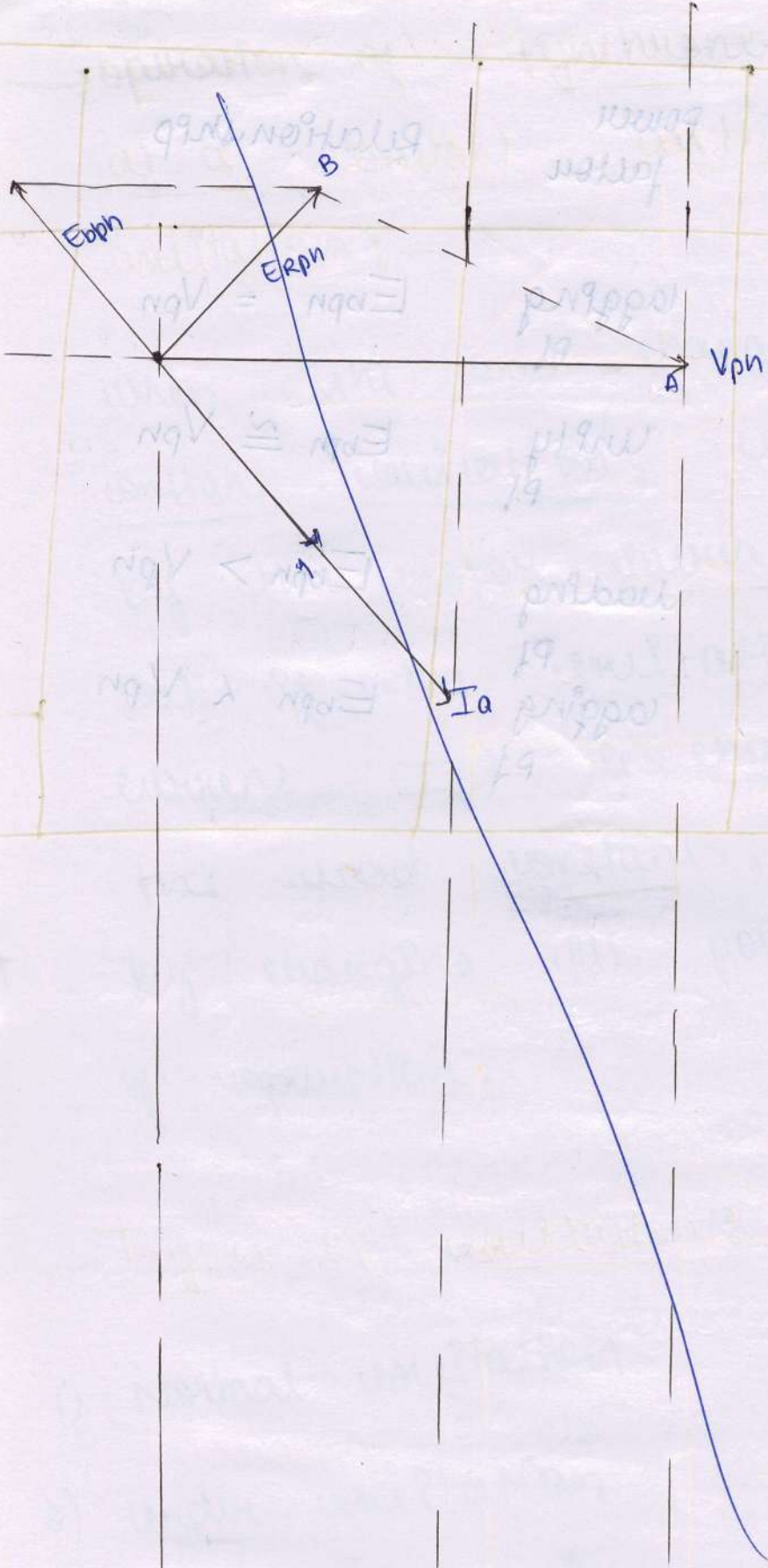


Sld. No.	Excitation	Power factor	Relationship
1)	Normal excitation	lagging P_f	$E_{bph} = V_{ph}$
2)	Critical excitation	unity P_f	$E_{bph} \approx V_{ph}$
3)	Over excitation	leading P_f	$E_{bph} > V_{ph}$
4)	Under excitation	lagging P_f	$E_{bph} < V_{ph}$

23/2

0/0
0/0

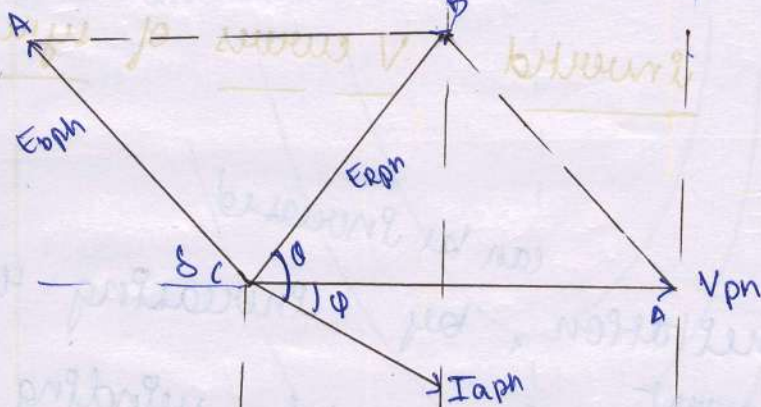
1) Normal
excitation



constant I_a lies ϕ line

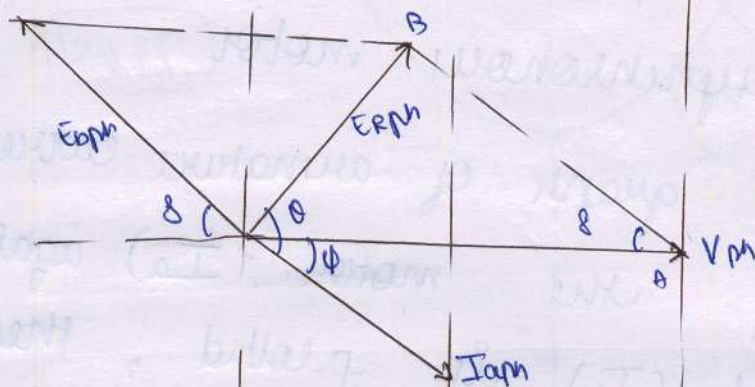
constant V_{ph} line

$1000/2/\sqrt{3}$



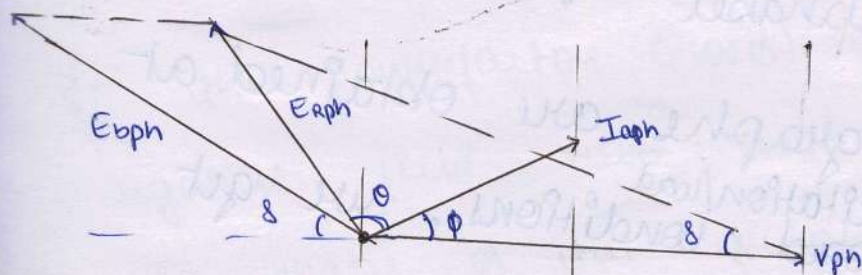
1) Normal excitation line

$E_{ph} = V_{ph}$
lagging pf
 I_a increases



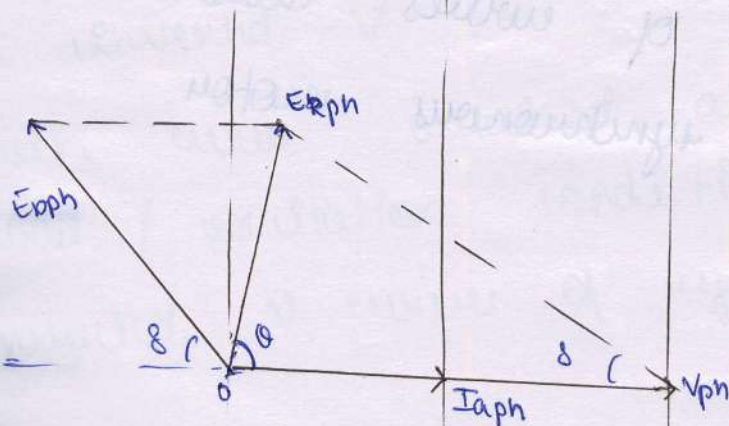
2) under excitation line

$|E_{ph}| < |V_{ph}|$
(ϕ is greater here)
 I_a increases



3) over excitation

$|E_{ph}| > |V_{ph}|$
(ϕ is less here)
 I_a



4) critical excitation

$E_{ph} \cong V_{ph}$

unity power factor

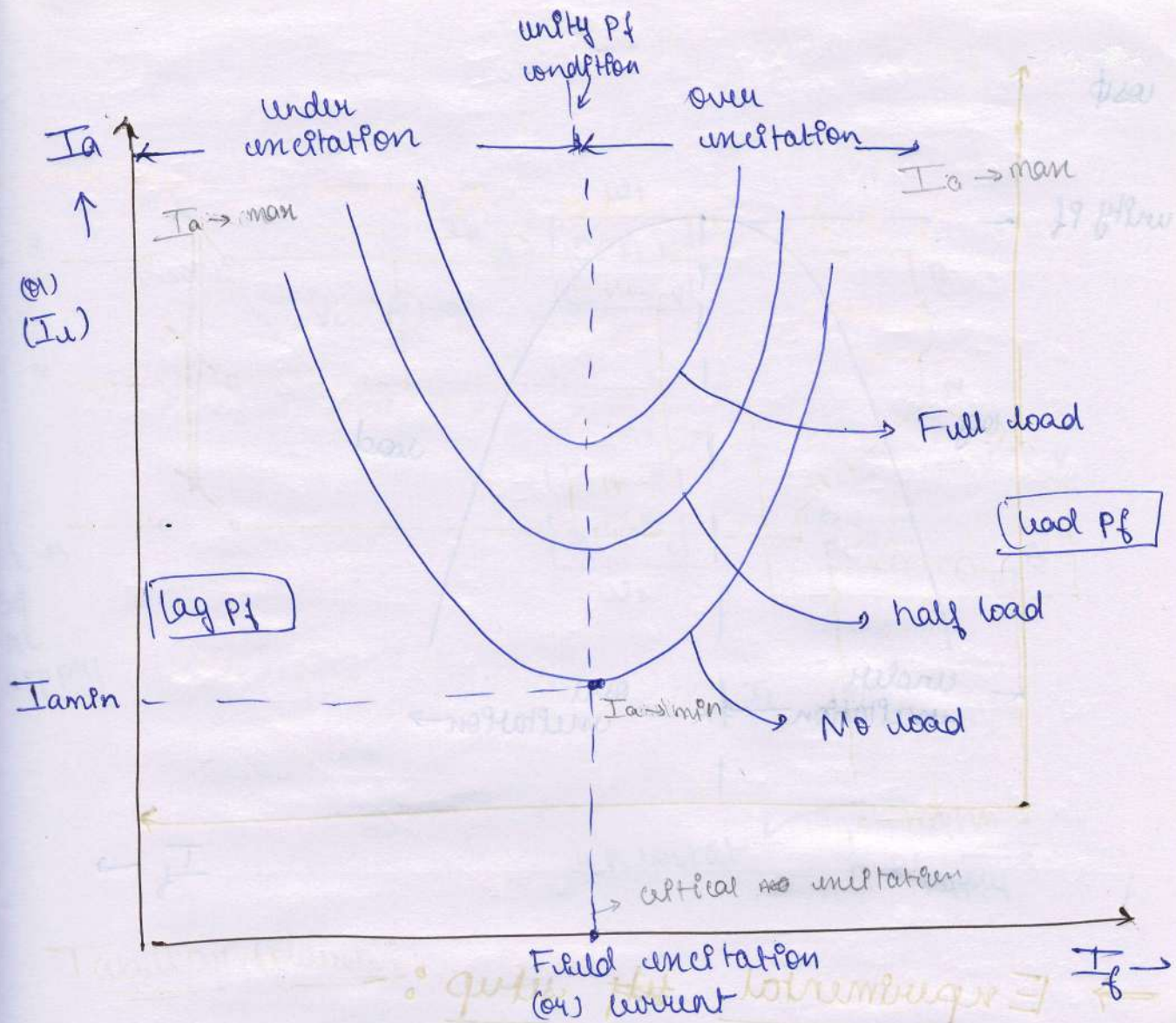
I_a is less/decreases

→ V and inverted V curves of synchronous motor:-

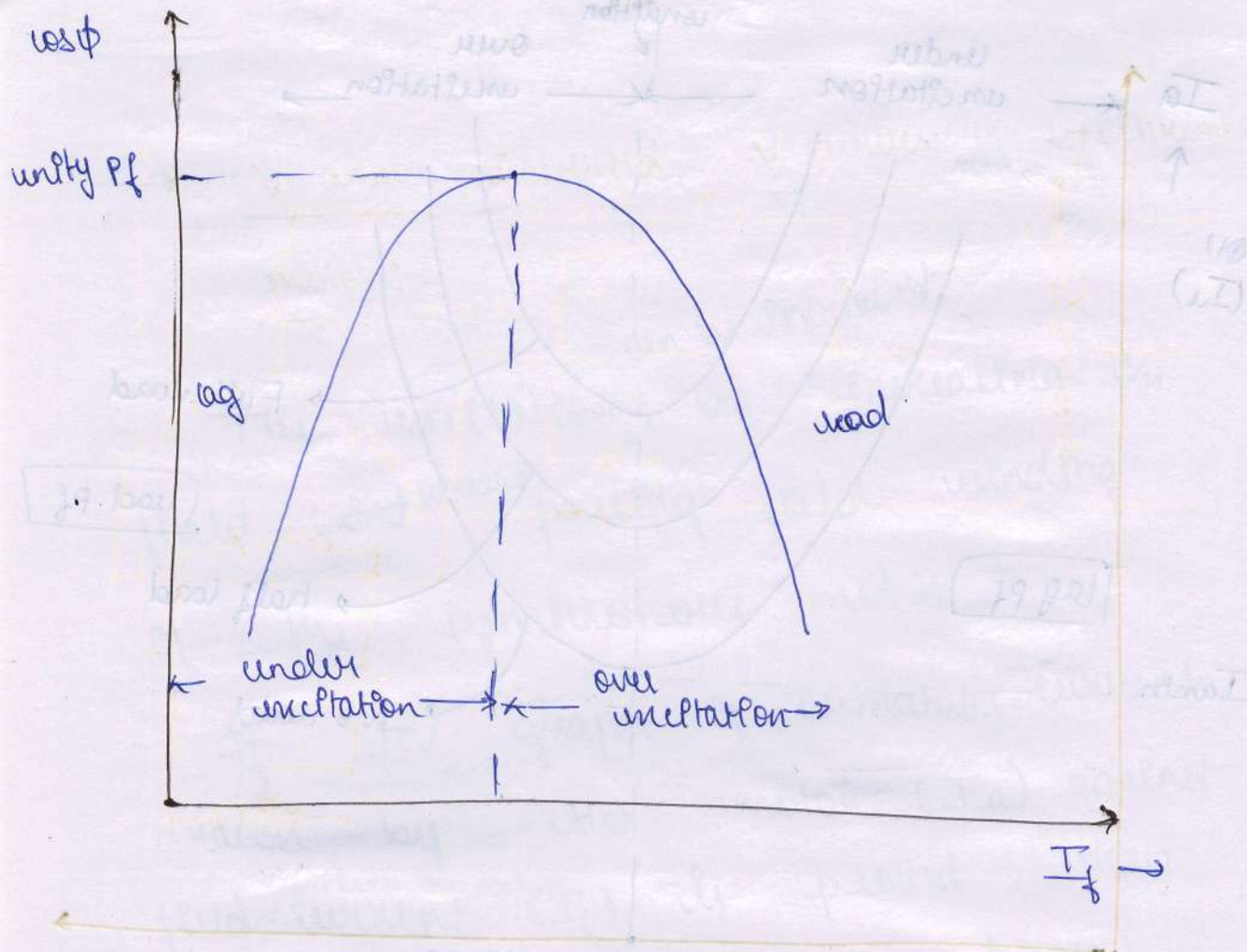
- The excitation, by increasing the field ~~current~~ passing field winding of the synchronous motor can be increased

- If the graph of armature current drawn by the motor (I_a) against field current (I_f) is plotted, then its shape looks like an english alphabet 'V'

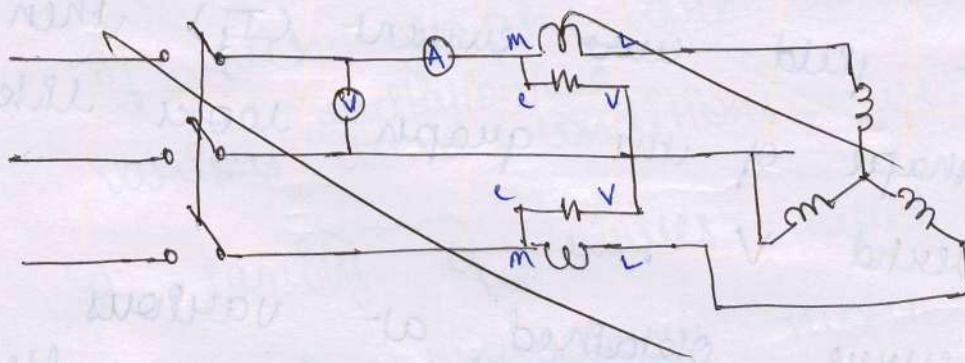
- If such graphs are obtained at various ~~load~~ excitation/load conditions, we get a family of curves. called V curves of synchronous motor

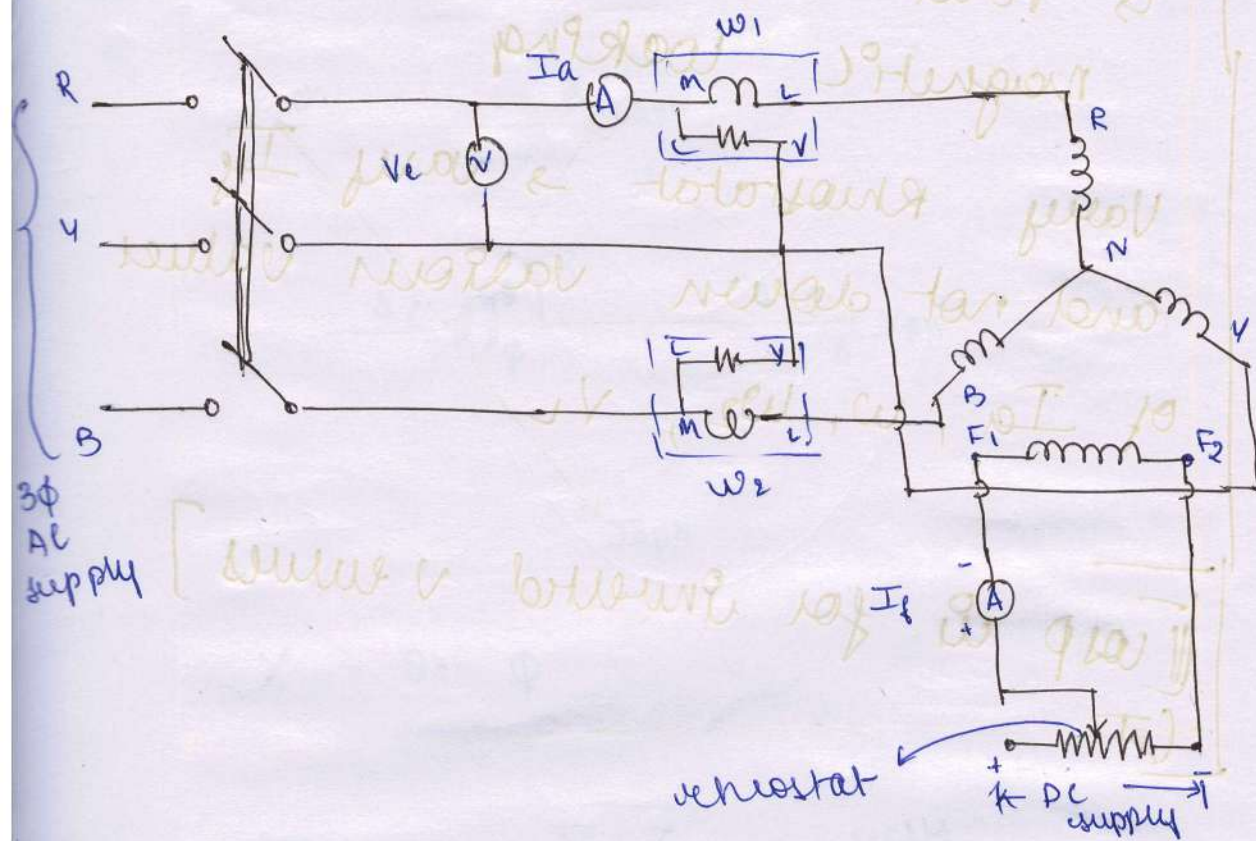


If the power factor ($\cos \phi$) is plotted against field current (I_f) then the shape of the graph looks like an inverted V. Such curves obtained at various load / excitation conditions are called inverted V curves of synchronous motor.



⇒ Experimental setup :-





Tabular column :-

Sl. NO.	V_e (volts)	I_a (Amps)	I_f (Amps)	w_1 (watts)	w_2 (watts)	$\cos \phi$ $\cos \tan^{-1} \left(\frac{\sqrt{3} (w_1 - w_2)}{w_1 + w_2} \right)$

$$\cos \phi = \cos \left[\tan^{-1} \frac{\sqrt{3} (w_1 - w_2)}{w_1 + w_2} \right] \quad \left[\phi = \tan^{-1} \frac{\sqrt{3} (w_1 - w_2)}{w_1 + w_2} \right]$$

→ Switch on AC supply & then use
(poney, damper, DC motor, 1φ IM, synchronous, DC machine) motor
once motor starts, install supply
DC field

poles are created

↳ pole $N_1 S_1, N_2 S_2$

magnetic locking

Vary Rheostat $\rightarrow$ vary I_f
and not down various values

of $I_a, \omega, \omega_2, V_L$

[Wsp is for inverted values]

$\Rightarrow$ Expression for back emf (E_{bph})

• $\rightarrow$ 1) For under excitation

$$E_{bph} < V_{ph}$$

$$Z_s = R_a + jX_s \quad \Omega/ph$$

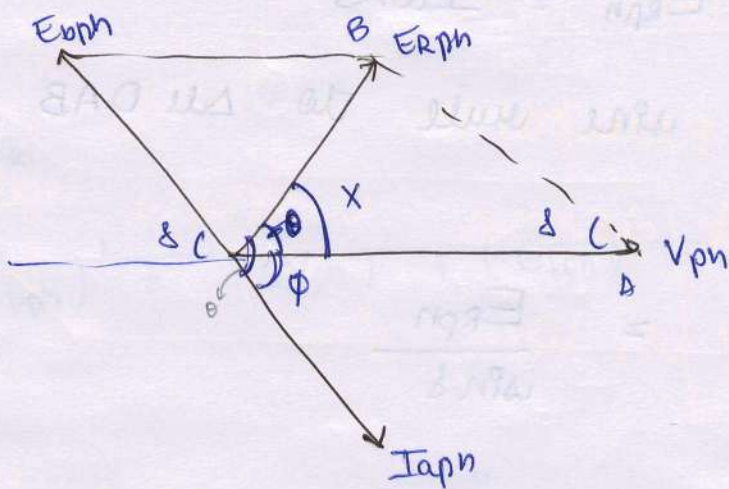
(or)

$$Z_s = |Z_s| \angle \theta$$

Ω/ph

where

$$\theta = \tan^{-1} \left(\frac{X_s}{R_a} \right)$$



$$X = \theta - \phi$$

$$E_{Rph} = I_a Z_s \quad \text{volts}$$

$$E_{Rph}$$

angle b/w E_{Rph} and $V_{ph} = \theta$

angle b/w V_{ph} and $I_{ph} = \phi$

applying cosine rule to triangle OAB

$$(E_{bp-ph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2 V_{ph} E_{Rph} \cos [\text{angle b/w } V_{ph} \text{ \& } E_{Rph}]$$

$$(E_{bp-ph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2 V_{ph} E_{Rph} \cos (X)$$

$$(E_{bph})^2 = (V_{ph})^2 + (E_{rph})^2 - 2 V_{ph} E_{rph} \cos(\theta - \phi)$$

where $\overline{E_{rph}} = \overline{I_a Z_s}$

Applying

sine rule to ΔOAB

$$\frac{E_{rph}}{\sin x} = \frac{E_{rph}}{\sin \delta}$$

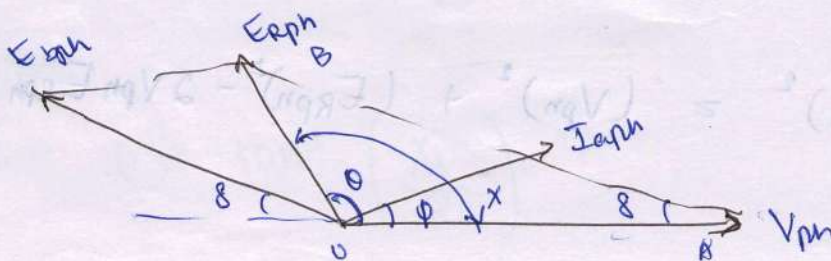
i.e.,

$$\frac{E_{bph}}{\sin(\theta - \phi)} = \frac{E_{rph}}{\sin \delta}$$

$$\sin \delta = \frac{E_{rph} \times \sin(\theta - \phi)}{E_{bph}}$$

• $\rightarrow$ 2) over excitation :-

$$E_{bph} > V_{ph}$$



$$X = \theta + \phi$$

→ Applying cosine rule for $\triangle OAB$

$$E_{\text{bph}}$$

$$(E_{\text{bph}})^2 = (V_{\text{ph}})^2 + (E_{\text{rph}})^2 - 2 V_{\text{ph}} E_{\text{rph}} \cos(\text{angle b/w } V_{\text{ph}} \text{ \& } E_{\text{rph}})$$

$$(E_{\text{bph}})^2 = (V_{\text{ph}})^2 + (E_{\text{rph}})^2$$

$$- 2 V_{\text{ph}} E_{\text{rph}} \cos(\text{angle b/w } V_{\text{ph}} \text{ \& } E_{\text{rph}})$$

$$(E_{\text{bph}})^2 = (V_{\text{ph}})^2 + (E_{\text{rph}})^2 - 2 V_{\text{ph}} E_{\text{rph}} \cos(X)$$

$$(E_{\text{ph}})^2 = (V_{\text{ph}})^2 + (E_{\text{rph}})^2 - 2 V_{\text{ph}} E_{\text{rph}} \cos(\theta + \phi)$$

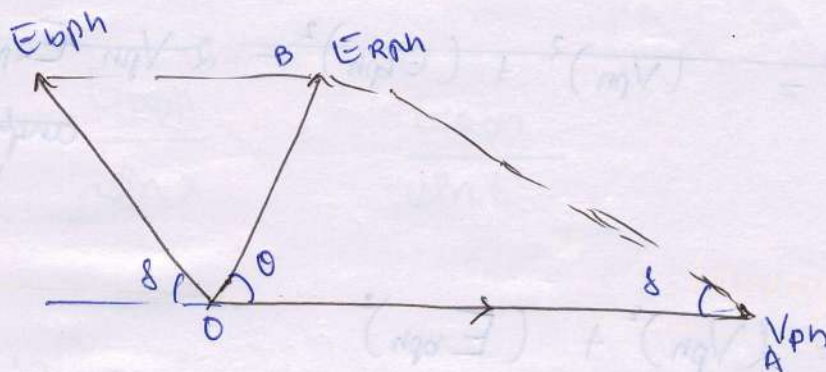
→ Applying sine rule to $\triangle OAB$

$$\frac{E_{\text{bph}}}{\sin X} = \frac{E_{\text{rph}}}{\sin \delta}$$

$$\sin \delta = \frac{E_{\text{rph}} \times \sin(\theta + \phi)}{E_{\text{ph}}}$$

↳ 3) critical mitigation :-

$$E_{bph} \approx V_{ph}$$



$$X = 0$$

$$\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix} \right] \phi = 0$$

→ Apply cosine rule

$$(E_{bph})^2 = (V_{ph})^2 + (E_{rph})^2 -$$

$2 V_{ph} E_{rph} \cos(\text{angle b/w } E_{rph} \text{ and } V_{ph})$

$\vec{E}_p$

$$(E_{bph})^2 = (V_{ph})^2 + (E_{rph})^2 - 2 V_{ph} E_{rph} \cos(X)$$

$$(E_{bph})^2 = \underline{(V_{ph})^2 + (E_{rph})^2 - 2 V_{ph} E_{rph} \cos \theta}$$

→ Apply sine rule

$$\frac{E_{bph}}{\sin \theta} = \frac{E_{rph}}{\sin \delta}$$

$$\sin \delta = \frac{E_{rph} \times \sin \theta}{E_{bph}}$$

→ In general

$$(E_{bph})^2 = (V_{ph})^2 + (E_{rph})^2 - 2 V_{ph} E_{rph} \cos(\theta \pm \phi)$$

+ → over excitation → leading Pf
- → under excitation → lagging Pf

Q1) A 2300 V, 3 ϕ , star connected SM has a resistance of $0.2 \Omega / ph$ and a synchronous reactance of $2.2 \Omega / ph$. The motor is operating at 0.5 Pf lead with a line current of 200 A. Determine the value of generated emf per phase.

Soln

$$V_L = 2300 \text{ V}$$

$$R_a = 0.2 \text{ } \Omega / \text{ph}$$

$$X_s = 2.2 \text{ } \Omega / \text{ph}$$

$$Z_s = R_a + jX_s$$

$$= 0.2 + j2.2$$

$$\cos \phi = 0.5 \text{ , load}$$

$$I_L = 200 \text{ A} = I_a$$

$E_{bph} = ?$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{2300}{\sqrt{3}}$$

$$= 1327.90 \text{ V}$$

$$I_{ph} = I_L$$

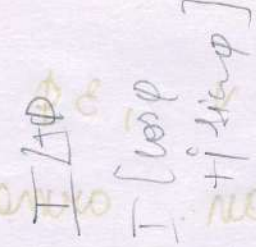
$$Z_s = 0.2 + j2.2$$

$$= 2.2 \angle 84.0^\circ \text{ } \Omega / \text{ph}$$

$$|Z_s| \angle \theta$$

$$\theta = 84.0$$

$\cos \phi = 0.5$
 $\phi = \cos^{-1}(0.5)$



$\phi = 60^\circ$

star connect
 $I_{apn} = I_L$

$E_{Rpn} = I_a Z_s$

Delta connected
 $I_{pn} = \frac{I_L}{\sqrt{3}}$

$= 200 \times 2.2$

$= 440 \text{ volts}$

$$\rightarrow (E_{bpn})^2 = (V_{pn})^2 + (E_{Rpn})^2 - 2 V_{pn} E_{Rpn} \cos(\theta + \phi)$$

$$= (1327.9)^2 + (440)^2 - 2 (1327.9)(440) \times \cos(84.8 + 60)$$

$E_{pn} = \sqrt{2791387.77 - 291794.71}$

$E_{pn} = 1670.74 \text{ volts}$

$E_{pn} = 1706.39 \text{ volts}$

$V_{pn} = V$
 $Z_s = R_s + jX_s$
 $Z_s = R_s + jX_s$
 $E_{Rpn} = I_a Z_s$
 $E_{bpn} = E_{Rpn} + V_{pn} \cos \phi$
 $0.826 = \cos \phi$
 35.2°

Q2) A 400 V, 3ϕ , star connected SM has an armature resistance of $0.2\ \Omega/\text{ph}$ and synchronous reactance of $2.0\ \Omega/\text{ph}$, while driving a certain load, it takes 25 A from supply. Calculate the back emf induced if the motor is working at

(i) $0.8\ \text{lag} \rightarrow 200.5\ \text{V}$

(ii) $0.9\ \text{lead} \rightarrow 252.5\ \text{V}$

(iii) $\text{UPF} \rightarrow 231.38\ \text{V}$

Soln $Z_s = 0.2 + j2$

~~Z_s~~ =

$Z_s = 2 \angle 84.28^\circ$

$= |Z_s| \angle \theta$

$\theta = 84.28^\circ$

$\rightarrow \cos \phi_1 = 0.8$

$\phi_1 = 36.86^\circ$

$\rightarrow \cos \phi_2 = 0.9$

$\phi_2 = 25.84^\circ$

$\rightarrow \phi_3 = 0$

$$E_{Bph} = I_a Z_s$$

$$= 25 \times 2$$

$$= 50 \text{ V}$$

$$(E_{Bph})^2 = \left(\frac{400}{\sqrt{3}}\right)^2 + (50)^2 - 2 \times \frac{400}{\sqrt{3}} \times 50$$

$$\times \cos (84.28 - 36.86)$$

$$E_{Bph1} = 200.5 \text{ V}$$

$$(E_{Bph})^2 = (230.94)^2 + (50)^2 - 2 \times 230.94 \times 50$$

$$\times \cos (84.28 + 25.84)$$

$$E_{Bph2} = 252.54 \text{ V}$$

$$(E_{Bph})^2 = (230.94)^2 + (50)^2 - 2 \times 230 \times 50$$

$$\times \cos (84.28)$$

$$E_{Bph3} = 231.38 \text{ V}$$

24/5/2017

Hunting in synchronous motor :-

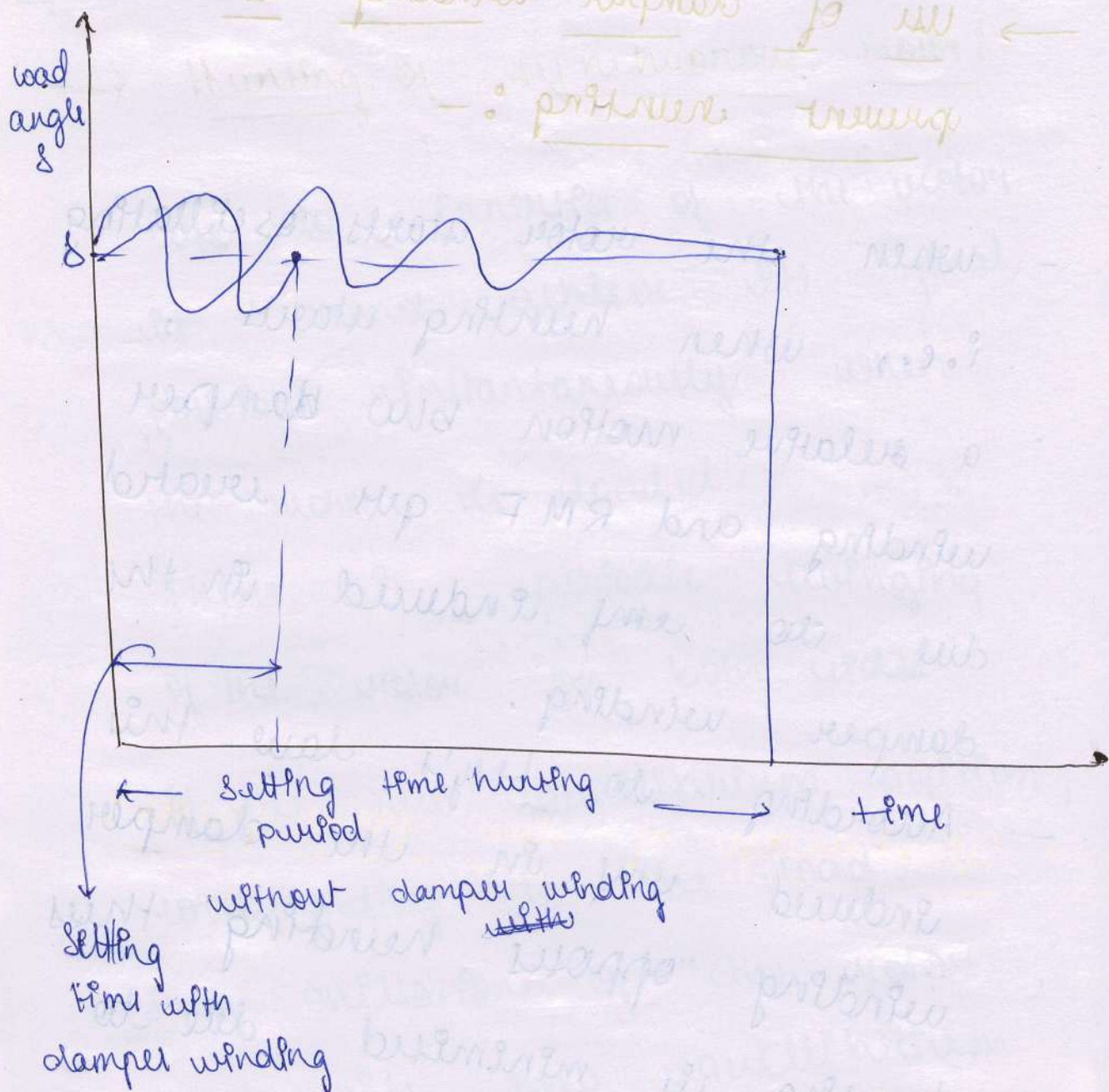
→ Hunting of synchronous motor :-

- Due to inertia of the motor it cannot achieve its final position instantaneously when the motor is loaded.
- There is periodic swinging of the motor on both sides of the new equilibrium position, corresponding to the load.
Such oscillations of the motor about its new equilibrium position, due to sudden application or removal of load is called swinging or hunting in synchronous motor.

→ Use of damper winding to
prevent hunting :-

- when the motor starts oscillating
i.e., when hunting starts,
a relative motion b/w damper
winding and RMF gets created
due to emf induced in the
damper winding.

- According to Lenz's law this
induced emf in the damper
winding opposes hunting thus
hunting is minimised due to
damper winding.



⇒ Synchronous condensers :-

- A synchronous motor running on no load when over excited ($E_{\phi ph} > V_{ph}$) is called a synchronous condenser or synchronous capacitor

- Under such condition, the current I_A will lead the voltage (V_{pn}) ~~at~~ by an angle almost equal to 90° .

- This is the property due to which synchronous motor are used as power factor improvement device (or) phase advancers.

→ Disadvantages of low power factor:-

* Single phase power

$$P = V I \cos \phi$$

$$I = \frac{P}{V \cos \phi}$$

- Let $P = 5 \text{ kW}$ and $V = 230 \text{ V}$
consider

~~Watt~~ $\phi \rightarrow$

(a) (i) :-

$$P_f = 0.8$$

(a) (ii) :-

$$P_f = 0.6$$

voltage regulation $\frac{1}{\text{efficiency}}$

$$(i) \quad \underline{I} = \frac{5 \times 10^3}{230 \times 0.8}$$

$$= \underline{27.17 \text{ A}}$$

$$(ii) \quad \underline{I} = \frac{5 \times 10^3}{230 \times 0.6}$$

$$= 8$$

$$= \underline{36.23 \text{ A}}$$

for any machine
P_f should be high
so current drawn
by the motor
should be less

* High current due to low P_f has the following disadvantages -

* The conductor size should be more increase which increases the cost

* More copper losses and less efficiency

* large current causes large voltage drop in transmission lines and this results in poor regulation

*

$$VI \cos \phi = \text{Active power}$$

$$\cos \phi = \frac{\text{Active power}}{VI}$$

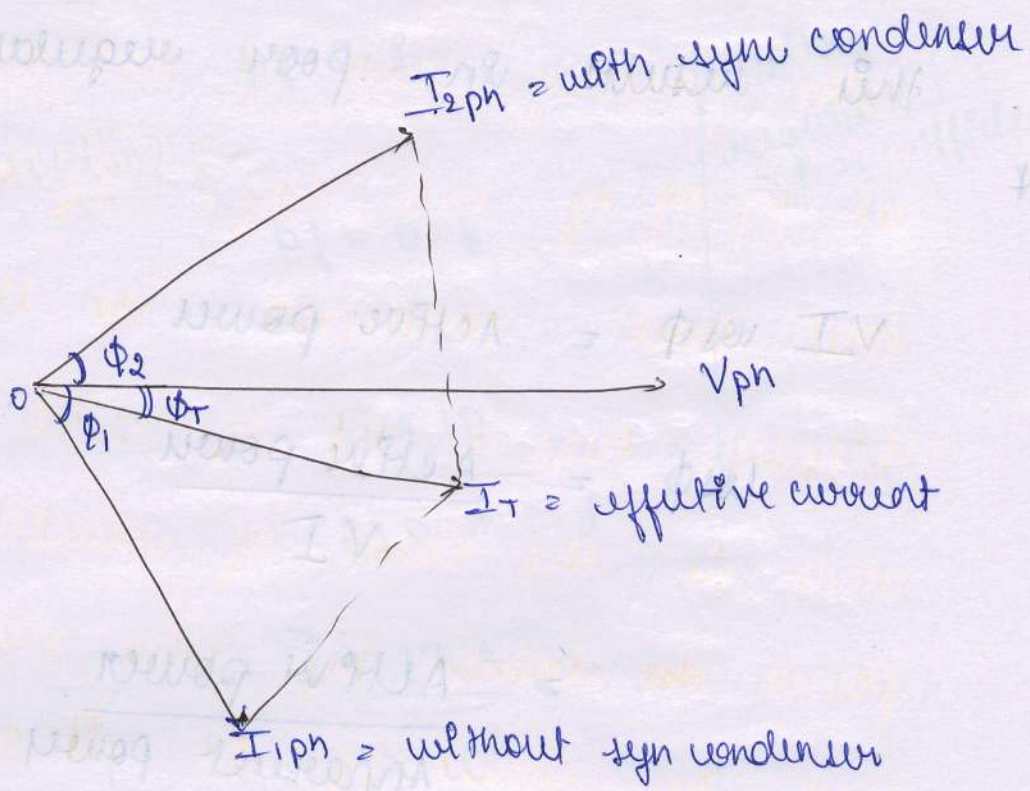
$$= \frac{\text{Active power}}{\text{Apparent power}}$$

$$\text{pf } \cos \phi = \frac{\text{KW}}{\text{KVA}}$$

If the pf is low then KVA rating of the equipment is high which increases the cost.

→ use of synchronous condenser in power factor improvement -

Q.1)



as $\phi_+ \downarrow \rightarrow \cos \phi_T \uparrow \rightarrow Pf \uparrow$

soln

Explanation
(over)

Note :-

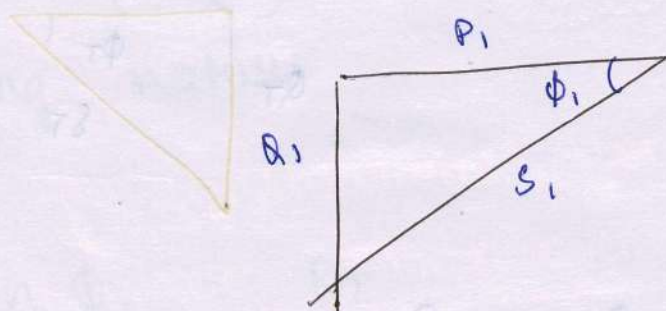
Such Pf correction equipment
must (syn condensers) be connected
across the supply

Q.1) A synchronous motor developing 20 kW is connected in parallel with factory load of 200 kW at p.f. of 0.8 lag. If the total load connected to the supply has a p.f. of 0.92 lag what is the value of reactive power ~~to~~ drawn by the motor and at what p.f. is it operating?

Soln

Given:-

load 1 = factory load 200 kW
 $\cos \phi_1 = 0.8 \text{ lag}$



Soln

$$\cos \phi_1 = 0.8$$

$$\phi_1 = 36.86^\circ$$

prigaleneb rotar marantunys A (1.8)

$$\tan \phi_1 = \frac{Q_1}{P_1}$$

$$Q_1 = P_1 \tan \phi_1$$

$$= 200 \times 10^3 \times \tan 36.8^\circ$$

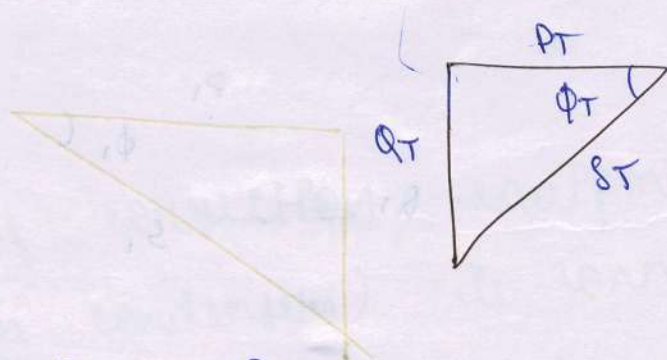
$$Q_1 = \underline{150 \text{ KVAR}} \quad (\text{Reactive power drawn by factory load})$$

for sync motor

$$P_2 = 20 \text{ KW}$$

$$\cos \phi_T = 0.92$$

$$\phi_T = 23.07^\circ$$



$$P_T = P_1 + P_2$$

$$= 200 + 20$$

$$= \underline{220 \text{ KW}}$$

$$Q_T = P_T \tan \phi_T$$

$$= 220 \times 10^3 \times \tan 23.07$$

$$Q_T = 93.70 \text{ KVAR}$$

Factor ~~symmetrical~~ $\rightarrow$ symmetrical

$$Q_1 + Q_2 = Q_T$$

$$Q_2 = Q_T - Q_1$$

$$= 93.7 - 150$$

$$Q_2 = -56.3 \text{ KVAR}$$



∴ -ve sign
indicates
leading power

The reactive power drawn by the motor is ~~56.3~~ -56.3 KVAR and -ve sign indicates its leading nature

$$\tan \phi_2 = \frac{Q_2}{P_2}$$

$$\phi_2 = \tan^{-1} \left(\frac{Q_2}{P_2} \right)$$

$$= \tan^{-1} \left(\frac{-56.3 \text{ kV}}{20 \text{ k}} \right)$$

$$\phi_2 = \underline{\underline{-70.44^\circ}}$$

$$\therefore \cos \phi_2 = \cos (-70.44^\circ)$$

$$= \underline{\underline{0.33}} \text{ leading}$$

Brondel diagram

BD ~~is~~ of a SM is an extension of simple phasor of ~~SM synchronous motor simple~~ for synchronous motor phasor

$$P_{in} = V_{ph} I_{ph} \cos \phi$$

$$\text{Copper loss} = (I_{aph})^2 R_a$$

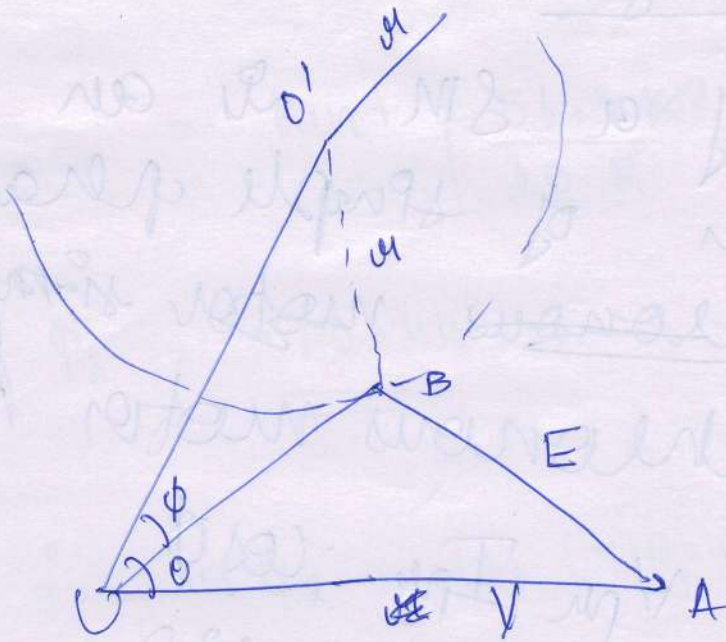
$$P_m = P_{in} - P_{cu}$$

$$P_m = V I \cos \phi - I_a^2 R_a$$

$$I^2 R_a - V I \cos \phi + P_m = 0$$

$$I^2 - \frac{V I \cos \phi}{R_a} + \frac{P_m}{R_a} = 0 \quad \text{--- (1)}$$

eqn (1) represents polar eqn of a circle to which, and draw a line at angle ϕ



The circle drawn with centre at O' and radius at $O'B$ represents circle of constant power

This is called constant power.
This is called Blondel diagram

- if excitation is varied while the power is kept constant then ~~work done~~ working point B while moving along the circle of constant power

$$P_{\text{induced}} = \frac{V^2}{4R_0}$$